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and experimentation in dynamic agency

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Abstract

The dynamics of a stochastic, two–period principal–agent relationship is studied. The agent’s type remains the same over time. Contracts are short term. The principal designs the second contract, taking the information available about the agent after the first period into account. Compared to deterministic environments significant changes emerge: First, fully separating contracts are optimal. Second, the principal has two opposing incentives when designing contracts: the principal ‘experiments,’ making signals more informative; yet dampens signals, thereby reducing up–front payments. As a result, ‘good’ agents’ targets are ratcheted over time.

Keywords and Phrases: Bayesian learning, Experimentation, Dynamic agency, Ratchet effect, Regulation, Procurement.

JEL Classification Numbers: D8, L5, H57.

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The simple analytics of information and experimentation in dynamic agency⋆

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Summary. The dynamics of a stochastic, two–period principal–agent relationship is studied. The agent’s type remains the same over time. Contracts are short term. The principal designs the second contract, taking the information available about the agent after the first period into account.

Compared to deterministic environments significant changes emerge: First, fully separating contracts are optimal. Second, the principal has two opposing incentives when designing contracts: the principal ‘experiments,’ making signals more informative; yet dampens signals, thereby reducing up–front payments. As a result, ‘good’ agents’ targets are ratcheted over time.

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1 Introduction

The underlying principle of competitive models is that all information gets disseminated through market interactions. However, the informational requirements for this result to hold are enormous. In the real world there are information asymmetries in the market and between agents. These asymmetries are used to

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explain competitive market failures. Indeed, asymmetric information is the key in many models for understanding anomalies observed in the real world. For example, in static agency models (with two types of agents) the main issue is how the principal can design a separating contract in which agents of different types self-select. In the process, these agents reveal their information. The procedure involves paying an informational rent to the ‘good’ agent. The question then naturally arises if such a procedure also works in a repeated or dynamic context. This question is dealt with by Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1987, 1993). They show that to get information revelation for a separating equilibrium in a dynamic context, an additional up-front payment is necessary. This additional payment stems from the possibility that the ‘good’ agent may attempt to deceive the principal in order to preserve future informational rents. In particular, he can deceive the principal by behaving in the first period like the ‘bad’ agent and subsequently profit from such deception in the second period. They show that, given the additional up-front payment, in a separating equilibrium, the contract imposed by the principal induces the same actions for the agent as in the static case.

At issue in all of these settings is the dissemination of information in the course of the interaction between principals and agents, and how informational rents and, hence, incentives are affected. Specifically, ‘good’ agents who have private information about their situation, accrue informational rents at the expense of the principal so long as their information remains private. As a consequence, whenever the duration of the contracts are shorter than the interaction between the parties, the principal has an incentive to design contracts in order to gather information. Conversely, agents have an incentive to attempt to deceive the principal in order to preserve informational rents.

The deterministic analysis for the dynamic case, as done by Freixas, Guesnerie and Tirole and Laffont and Tirole, leaves many issues unsettled. In particular, in a separating equilibrium, after the up-front payments are made, the agent’s type is revealed. The principal takes advantage of this information in the second period by reducing the rents of all the agents to zero. In many if not most instances of agency, however, it is natural to expect that the principal cannot directly infer the actions of the agents. In this case, slow learning rather than complete learning is implied. In order to deal with these issues, Jeitschko and Mirman (JM) (2002) introduce noise in a general agency model, so that the principal cannot directly infer the actions of the agent in the first period. This gives rise to several informational effects. JM show that in accounting for these effects when determining the equilibrium targets, the principal considers two opposing interests.

First, the principal desires to learn the agent’s private information for use in designing future contracts. Specifically, the principal manipulates the information generating process by the choice of the agent’s equilibrium actions to enhance the information inferred from the signal. That is, the principal experiments. Second, in order to induce the agent to take the equilibrium actions, the principal accounts for the agent’s dynamic incentives. Since the principal cannot commit
to a long term contract, he must account for learning and the implied reduction of future informational rents of the agent. Consequently, as in the deterministic case, a good agent is given an additional up–front payment in equilibrium. The amount of the up–front payment depends on two things. First, the degree to which future informational rents are sustained in equilibrium. And second, how much an agent can gain by deceiving the principal. Thus, the additional up–front payment is equal to the agent’s discounted gain from deception over truth–telling. This difference depends on the informativeness of the signal that the principal observes. In particular, the less informative the signal, the smaller the up–front payment. Since the degree of information transmission is determined by the equilibrium targets, the principal chooses targets to reduce the up–front payment, in a sense, committing himself not to learn too much too fast. We refer to this as signal dampening.

In this paper, we deal with a problem similar to the one in JM. However, we do it in a slightly simpler context. In JM, a general distribution of the noise is modeled. Under this assumption the problem can be studied in its most general form. In this paper, we deal with a uniform distribution. This gives us the ability to find closed form solutions. Thus, although our results are similar to the general case of JM, we are able to get, in this paper, more focused and precise results. Specifically, due to the simplification, we are able to produce, in a more precise way, conditions under which (only) a separating equilibrium is optimal. Moreover, the choices of the agent are now in closed form. This allows us to interpret the results in a clear, intuitive and unambiguous way. For example, we are able to study the trade off, found in JM, between desiring more noise to reduce the up–front payment and for less noise in order to learn for use in the second period. We show, in this case, that the desire to reduce the up–front payment dominates, i.e., outcomes produce more noise then in the corresponding static problem. We are also able to find an exact, simple and intuitive contract specifying the payment to the agents as a function of the random output. In particular our contract specifies a two tiered payoff, so that low outcomes yield a small payment and for high outcomes a fixed supplement is added.

We address these issues in a simple model of repeated procurement with stochastic production in which there are two possible types of agent. As mentioned above, due to the path-breaking works especially by Laffont and Tirole, this type of interaction is well–understood when production is deterministic. However, the addition of noise in the environment leads to substantial changes when compared to deterministic settings. To be sure, there are two fundamental differences. The first concerns the type of equilibrium that results in stochastic environments. The second is how the flow of information in the course of the interaction is affected, indeed manipulated, by the equilibrium actions set by the principal.

A consequence of the immediate and complete learning in deterministic environments when agents’ actions reveal their types is that fully separating contracts are hard to support. Specifically, in a deterministic setting it is well–known that the two types of agent may not be induced to target distinct signals in the first
period unless the discount factor is sufficiently low. That is, if agents value the future highly, the first period contract has the same actions taken by the agent regardless of his type in a pooling or semi-separating equilibrium.

This is so, because when the two different types produce distinct outputs in the first period, the principal learns the agent’s type. If the agent is the ‘good’ type, the principle infers this upon observing the equilibrium signal and extracts all informational rents that otherwise accrue to the good type. However, if the good agent mimics the ‘bad’ agent, the principal is deceived and lead to believe that the agent is the bad agent. In this case the principal offers the first-best contract for the bad agent and the good agent receives substantial informational rents. Thus, in order to prevent the good agent from deceiving the principal, he must be compensated by giving him the additional up-front payment, mentioned above. This up-front payment is equal to the good type’s discounted gain from deception over truth telling.

These payments may be quite substantial, unless the agent discounts the future heavily. In fact, they can easily be so large that a low type agent is better off mimicking the high type agent in the first period in order to obtain this payment, and then terminating the relationship with the principal. This is referred to as the take-the-money-and-run strategy.

In instances in which agents do not discount the future sufficiently the principal is only able to design contracts that pool the agents’ signals in a pooling or semi-separating equilibrium with mixed strategies. These types of equilibrium contracts impede the flow of information from one period to the next, because the agent is instructed to randomize his actions. A somewhat unappealing feature of such equilibrium contracts is that the agent has no particular reason to follow the principal’s mixing instructions, and the principal cannot actually verify whether or not the instructions are in fact followed.

In a stochastic environment this problem does not arise. Recall that the amount of the up-front payment made to the good type is the discounted difference in informational rents that he would obtain in the future if he deceives the principal instead of taking the equilibrium action. If the interaction is sufficiently noisy, the principal’s ability to learn the agent’s type is impeded, and therefore, if the agent chooses the equilibrium action, the high type does not lose all informational rents in the second period. Moreover, due to noise, deception is not as effective if the agent mimics the bad type, since the principal may not infer that the agent is the low type. Hence, the up-front payment is smaller in noisy environments so that a fully separating equilibrium is optimal, and the principal does not induce inefficiencies through mixed strategies.

Thus, while both mixed strategies and noise in production impede the principal’s ability to learn, this should not be understood to mean that noise in production plays the same role as incomplete learning in a semi-separating contract in deterministic models and ‘replaces’ the importance of the principal’s choice of first period equilibrium actions in determining information transmission. On

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1 Indeed, Laffont and Tirole (1993) stress that the robust results in the deterministic two-type case hinge on the size of the discount factor.
the contrary, in a deterministic model the principal’s choice of targets are always the same—regardless of whether the contract has pure or mixed strategies—and, hence, the choice of targets do not affect the information transmission. However, in a stochastic environment, the equilibrium targets themselves are highly critical in affecting information transmission—indeed, the optimal first period contract is fully separating. Indeed, this is the second fundamental difference that noise introduces in the environment when compared to the deterministic setting; as shown in JM, the principal uses the choice of first period targets as a vehicle for manipulating the degree of information dissemination in the course of the interaction with the agent.

Indeed, as derived in this paper, the decision maker’s choice of actions, in order to manipulate the flow of information, has implications regarding the dynamics of equilibrium targets. Hence, the so-called ‘ratchet effect.’ The ratchet effect has a long history in short-term contracting under asymmetric information in both practical and theoretical economics. The term was first coined by Berliner in connection with Soviet planning. Thus,

“A certain universal planning practice [...] operates like a ratchet in the planning mechanism, so that once a new high level of performance has been achieved, the next plan target [...] must usually be raised above it. [...] The ratchet principal applies not only to production targets, but to the planning of profit and cost targets as well.”

It has long since been recognized that the ratchet effect occurs in a wide variety of settings, not just central planning. Indeed, any instance in which contracts are formed under asymmetric information and relationships last longer than contracts, can give rise to the ratchet effect. Thus, in government procurement and regulation, interactions are often longer term, whereas the governing agency cannot commit to long term contracts, often due to legal constraints. Moreover, in private and public bureaucracies and administrations, ‘contracts’ (e.g., budget allocations) are frequently of short duration (e.g., a fiscal quarter of year), even though the relationship between parties within the organization may be of much longer duration. Finally, there is an extensive literature on the ratchet effect in contracting between firms and suppliers as well as workers and management.

As a result of the manipulation of the flow of information, the original ratchet effect re-emerges as part of the equilibrium. That is, a high level of performance leads to an increasing of the target level. This, too, is in stark contrast to deterministic models, where if, in fact, a separating equilibrium can be supported, the principal chooses the initial targets to be the same as in a static setting. This yields a curious dynamics for the targets in the course of the interaction. Unlike the original ratchet effect as stated in Berliner (1957), the exact opposite effect takes hold in a deterministic setting. That is, if a high level of performance is achieved, the target remains unaffected, whereas if a low level of performance is achieved, the target is raised.

2 Berliner (1957, p. 78).
2 The model

Consider a two-period version of delegated production of which the deterministic static variant is outlined in Hurwicz and Shapiro (1978) and studied by Harris and Townsend (1981). Assume that contracts are short-term and newly designed at beginning of each period. That is, the principal has no power to commit to future contracts — a necessary condition for the ratchet effect to be a concern. Moreover, we suppose that production is affected by unobservable homoskedastic noise, as is first suggested in Salgueiro (1991).

In period $t$, $(t = 1, 2)$ an employer (the principal) hires a worker (the agent) to apply effort, denoted by $e_t \in \mathbb{R}_+$, to a production process. The production technique is

$$y_t = \theta e_t + \epsilon_t, \quad t = 1, 2.$$  \hspace{1cm} (1)

Here $\theta \in \{\underline{\theta}, \bar{\theta}\}$ is the (time-invariant) productivity parameter with $0 < \underline{\theta} < \bar{\theta}$. The productivity parameter may either reflect intrinsic abilities of the worker, or be a feature of the production technology available to the worker. The term $\epsilon_t$ denotes an unobservable random shock to output that is realized after the agent’s effort is applied. Assume that $\epsilon_t$ is distributed uniformly on an interval of length $2\eta$, centered around 0, and is independent of $e$ and $t$.

The agent knows the value of $\theta$. If the state of the world is such that the productivity parameter is $\bar{\theta}$ the agent is referred to as a ‘high type,’ otherwise the agent is referred to as a ‘low type.’ The agent has a reservation utility of 0. If, in period $t$, the agent exerts effort $e_t$ and receives a payment of $r_t$, then his utility is

$$u_t = r_t - e_t^2.$$  \hspace{1cm} (2)

The principal does not know the true value of the productivity parameter. At time $t$ she believes that $\theta = \underline{\theta}$, with probability $\rho_t$, and with probability $1 - \rho_t, \theta = \bar{\theta}$. The principal’s utility in period $t$ is given by

$$v_t = y_t - r_t.$$  \hspace{1cm} (3)

Output $y_t$ is observed by both the principal and the agent, but the principal is unable to observe the agent’s effort. Moreover, since $\epsilon_t$ is also unobservable, the principal can only draw inferences about the agent’s effort in period $t$. Note that both the principal and the agent are risk-neutral.

We suppose that the sole basis for rewarding the agent is the observed output, $y_t$. This assumption may appear restrictive in that it precludes additional messages or signals through which the agent reveals his type to the principal. However, due to the principal’s inability to commit to a long-term contract, restricting attention to contracts based only on observable output is, in fact, without loss of generality. In particular, the optimal contract in the larger space that may include additional messages remains based only on observable output, which is, in fact, the optimal equilibrium contract that we study.\(^3\)

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\(^3\) This argument is demonstrated formally in Corollary 1 below.
The sequence of events is as follows: at the outset the agent observes the true state of the world, i.e., the value of $\theta$. The principal offers a contract which consists of a reward schedule $r_1(y_1)$ specifying rewards to be paid to the agent based upon the observed output $y_1$. The agent either accepts the contract or rejects it. An agent who rejects the contract receives his reservation utility of 0 and the relationship is dissolved. If the agent accepts the contract, in period $t = 1$ he applies effort to the production technology. After effort has been applied the random shock $\epsilon_1$, and hence output $y_1$, are realized. Both the agent and the principal receive their first period payoffs.

At the end of the first period, using Bayes’ rule, the principal updates her beliefs regarding the state of the world, on the basis of observed output and knowledge of the first period contract. Using the updated beliefs, $\rho_2$, the principal designs a contract that specifies rewards based on second period output. The agent can accept or reject this contract. An agent who rejects the contract receives the reservation payoff of zero and the game ends. An agent that accepts the second period contract applies effort to the production process, after which $\epsilon_2$ and hence $y_2$ are realized. The principal and the agent receive their second period payoffs and the game ends.

The production process, payoff functions, distribution of noise in production, and the principal’s prior beliefs, $\rho_1$, are common knowledge, as is the fact that the principal uses Bayes’ rule in updating beliefs. Lastly, suppose that the principal’s and agent’s common discount factor is given by $\delta > 0$.

In the next Section the equilibrium targets are derived, and it is shown that the first period contract is fully separating. The impact of the principal’s two incentives to manipulate the flow of information by the choice of first period equilibrium actions are examined in Section 4, where it is shown that the high type’s target is ratcheted up in the course of the interaction with the principal.

Due to noise in production, in equilibrium, many distinct levels of output may be observed. Therefore, the equilibrium contracts are reward functions that map all possible (equilibrium) outputs into rewards. These contracts assure that the equilibrium effort levels implied by the targets derived in Sections 3.1 and 3.2 are induced by yielding the corresponding expected rewards for the two types of agents. This is done in Section 5, where it is shown that the equilibrium contracts for both periods consist of a simple base–pay and added bonus for high levels of output achieved.

3 The equilibrium targets

In this Section the equilibrium targets for both periods are derived. Since the model is solved through backward induction, we begin with the second period.

3.1 Second period target output levels

After the first period the principal uses the observation on first period output to update her beliefs about the agent’s type. The principal does so using Bayes’
rule. For completeness, should out-of-equilibrium observations occur, assume that the principal believes the agent to be the low type if output is lower than equilibrium levels of output and believes the agent is a high type if observations above equilibrium levels are made. Specifically, letting $\bar{y}_1$ denote the “high” target output of the first period contract, that is, the amount that the high type agent is instructed to produce in the first period, $y_1$ the “low” first period target output, and $y_1$ the observed output, that is, the output that results from the first period effort and the noise term, the principal’s belief function is given by:

**Lemma 1.** The principal’s posterior is given by

$$
\rho_2(y_1 | \bar{y}_1, y_1) = \begin{cases} 
0, & \text{if } y_1 \in (-\infty, \bar{y}_1 - \eta), \\
\rho_1, & \text{if } y_1 \in [\bar{y}_1 - \eta, \bar{y}_1 + \eta], \\
1, & \text{if } y_1 \in (\bar{y}_1 + \eta, \infty).
\end{cases}
$$

Thus, if the environment is sufficiently noisy (i.e., $\eta$ large), the principal’s belief function can take on three values. If output is observed that can only occur in one state of the world, the principal believes to be fully informed, that is, her beliefs are either 0 or 1. For realized output levels between these extremes the principal’s posterior coincides with her prior, since, conditioned on the state, it is equally likely that observed output would be in this interval.

Notice that the only argument of the belief function is observed output, not the agent’s actual actions, since the agent’s actions are unobservable. That is, the principal’s beliefs are solely determined by the output she observes, $y_1$, and the equilibrium expected levels of output, $\bar{y}_1$ and $y_1$. Of course, she designs the first period contract so that the agent chooses the equilibrium actions and therefore, in equilibrium, the principal’s beliefs coincide with objective probabilities.

For given posterior beliefs, $\rho_2$, the principal maximizes her expected utility by choosing two output levels, one for each possible type of agent, and an output–contingent reward to be paid to the agent upon realization of the output. This problem is essentially the static problem studied in Harris and Townsend (1981). The only difference is that since production is stochastic, the reward structure must account for a range of possible output levels. The shape of this reward structure is characterized in Section 5. For now we focus on the target output levels.

For the second period static problem it is known that target levels and rewards are chosen so that the “high” output is produced by the high type agent and the “low” output is produced by the low type. Moreover, the high type agent’s incentive compatibility constraint is binding in equilibrium, as is the low type’s individual rationality constraint. All other constraints are slack. That is, letting $(\bar{y}_2, r_2)$ denote the high target output and reward, and $(y_2, L_2)$ the low target output and reward, the principal chooses target outputs and rewards to maximize the expectation of

$$
v_2 = \rho_2(\bar{y}_2 - r_2) + (1 - \rho_2)(y_2 - L_2),
$$

s.t. $\bar{y}_2 - (\bar{y}_2/\theta)^2 = L_2 - (y_2/\theta)^2$, and

$$
(4)
$$
Here the first constraint is the high type’s incentive compatibility constraint and the second is the low type’s individual rationality constraint. The first order conditions are sufficient.

Substituting the binding constraints into the objective function, letting \( \Theta \equiv \frac{\theta}{\theta} \) and \( C_t \equiv \frac{1}{1-\rho_t} \), the first order conditions yield the equilibrium output targets. For given beliefs \( \rho_t \) these are

\[
\bar{y}_t = \frac{\theta_t}{2}, \quad \text{and} \quad y_t = C_t \frac{\theta_t}{2}.
\]

The corresponding rewards, \( r_2 \) and \( r_1 \) are implied by the binding constraints, (4) and (5), above.

If \( \rho_2 = 0 \), then the second period contract is the ‘full information’ optimal contract when it is known that the productivity parameter is \( \theta \). This yields \( C_2 = 1 \) and the “first best” output level, \( \frac{\theta_t}{2} \) for a low type agent, i.e., the amount of output the principal who believes the worker is a low type wants produced and the corresponding reward necessary to induce this effort level. This combination of expected output and reward is given by the point \( 2^* \) in the diagram. Notice that, for all \( \rho_2 > 0 \), \( C_2 < 1 \), so the optimal contract under ‘incomplete information’ has the low type agent producing less than the “first best” level of output (point \( 2 \) in the diagram). Notice that in both cases the agent is at his reservation level of utility, that is, both points, \( 2^* \) and \( 2 \), lie on the low agent’s reservation level indifference curve, denoted by \( \mu = 0 \) in the diagram.

Next suppose that \( \rho_2 = 1 \), i.e., the full information optimal contract when it is known that the worker is a high type. In this case, \( C_2 = 0 \) and the full information contract specifies \( \theta_t/2 \) — the “first best” output level for the high type agent, which will leave the high type at his reservation level of 0. This expected output and reward pair is given by point \( 2^* \) on the high type’s reservation level indifference curve denoted by \( \pi = 0 \) in the diagram.

Note that in the incomplete information contract (represented by the points \( 2 \) and \( 3 \), the high type’s target output, and hence the high type’s effort level, remains at the “first best” level, that is, \( \bar{y}_2 \) is independent of the principal’s posterior beliefs. However, in the incomplete information contract, due to the high type’s binding incentive compatibility constraint, to implement the equilibrium effort level, given by \( e_2 = \bar{y}/2 \), the principal has to increase the reward paid to the high type by \( C_2 \frac{\theta_t}{4} (1 - \Theta^2) \). This is the informational rent that the high type obtains due to the informational asymmetry. The implication of the binding incentive compatibility constraint in the diagram is that both points \( 2 \) and \( 3 \) are on the same indifference curve of the high type.

### 3.2 The first period target outputs

Since the productivity parameter remains unchanged from the first to the second period, the principal takes into account the inferences about \( \theta \) that can be drawn
from observing the first period output and the effects of these inferences on her second period payoff, when designing the first period contract. Thus, the principal’s problem is to choose \((y_1, r_1)\) and \((y_1', r_1')\) to maximize
\[
E(v_1 + v_2) = \rho_1(y_1 - r_1) + (1 - \rho_1)(y_1' - r_1') + \delta E\bar{v}_2(\rho_2(y_1|y_1, y_1')),
\]
subject to the incentive compatibility constraints and the individual rationality constraints of the two types.

Given the second period contract, the high type obtains positive informational rents in the second period if \(\rho_2 < 1\). Specifically, given the second period contract, the high type’s future informational rents are given by \(C_2^2(\hat{\theta}/4)(1 - \Theta^2)\), where \(C_2\) is a function of \(\rho_2\), strictly positive for all \(\rho_2 < 1\). Thus, the high type agent has an incentive to manipulate the principal’s beliefs in order to increase future rents. Clearly, the high type cannot change the principal’s belief function (Lemma 1). However, through his first period effort, he can influence first period output stochastically, and thus affect the principal’s beliefs governing the second period contract and rents.

In equilibrium, contracts are structured such that, due to the high type’s binding incentive compatibility constraint, the only first period output levels the high type could find optimal to target are \(y_1\) and \(y_1'\). That is, either he chooses the target output intended for him, or he mimics the low type agent, by targeting the low type’s equilibrium output — all other output targets can be ruled out by choice of the reward function.

If the high type targets the equilibrium output level \(y_1\), observed output can range from \(y_1 - \eta\) to \(y_1 + \eta\). Given the principal’s belief function (Lemma 1), this means that if \(\epsilon_1\) is large, the principal infers the agent’s type. If this happens, the agent loses all informational rents in the second period (point \(2^*\) in the diagram). However, if \(\epsilon_1\) is small, the principal obtains no new information and her posterior coincides with her prior \((\rho_2 = \rho_1)\). In this case the principal again designs an optimal contract accounting for both types of agent (points 2 and \(\bar{2}\)), and the high type agent gets an informational rent of \(C_2^1(\hat{\theta}/4)(1 - \Theta^2)\).

Now suppose the high type targets an output of \(y_1'\). Then observed output comes from the interval \([y_1' - \eta, y_1' + \eta]\). In this case, if \(\epsilon_1\) is large, the principal’s beliefs coincide with her prior, so the second period contract gives the high type the same informational rent as above when the principal cannot update her beliefs. However, if \(\epsilon_1\) is small, the principal will incorrectly infer that the agent is a low type. In response, she designs the first–best contract for a low type agent in the second period, point \(2^*\) in the diagram. Notice that this point lies above the high type’s indifference curve for the contract under incomplete information, in fact, the high type agent who has thus successfully deceived the principal obtains informational rents of \((\hat{\theta}/4)(1 - \Theta^2)\) in the second period.

Given the distribution of noise in production, for either target, the probability that the principal will obtain no relevant information, so that her posterior coincides with her prior is given by \(\left(\frac{y_1' - y_1 + 2\eta}{2\eta}\right)^2\). The complementary probability,
namely that the principal believes to be fully informed about the agent’s type, is hence given by \( \frac{T_1 - \nu_1}{2\eta} \).

Therefore, taking into account the discounting, the high type’s binding incentive compatibility constraint is given by

\[
\gamma_1 \left( \frac{y_1}{\beta} \right)^2 + \delta C_1^2(\sigma^2/4)(1 - \Theta^2) \left( \frac{y_1 - T_1}{2\eta} \right) = \gamma_1 - \left( \frac{y_1}{\beta} \right)^2 + \delta(\sigma^2/4)(1 - \Theta^2) \left( \frac{T_1 - \nu_1}{2\eta} \right),
\]

or, consolidating terms and solving for the high type’s first period expected reward,

\[
\gamma_1 = \left( \frac{y_1}{\beta} \right)^2 + \gamma_1 - \left( \frac{y_1}{\beta} \right)^2 + \delta(\sigma^2/4)(1 - \Theta^2) \left( \frac{T_1 - \nu_1}{2\eta} \right). \tag{9}
\]

Thus, compared to the static incentive compatibility constraint of the high type in the second period contract (cf. Equation (4)), in the first period the high type’s reward is greater by \( \delta(\sigma^2/4)(1 - \Theta^2) \left( \frac{T_1 - \nu_1}{2\eta} \right) \) in order to induce him to choose the equilibrium first period targeted output. This amount is exactly the discounted utility the agent stands to lose by choosing the high equilibrium target instead of the low target.

Due to this increased payment, as is well–known, the incentive compatibility constraint of the low type agent may become binding (resulting in the so–called take–the–money–and–run strategy). However, we proceed by using only the low type’s individual rationality constraint and later show that his incentive compatibility constraint is slack in sufficiently noisy environments.

Since the low type is kept at his reservation utility of 0 in the second period, the future has no impact on the low type and the problem is essentially static. Thus his individual rationality constraint is as before and given by

\[
\gamma_1 - \left( \frac{y_1}{\beta} \right)^2 = 0. \tag{10}
\]

To complete the analysis of the principal’s first period problem, given in (8), consider the principal’s expected second period payoff. Given the second period target outputs and rewards (see (6) and (7)), and the distribution of noise and the principal’s belief function (Lemma 1), this is

\[
Ev_2(\rho_2(y_1|\gamma_1, y_1)) = \left( \rho_1(\sigma^2/4) + (1 - \rho_1)(\sigma^2/4) \right) \left( \frac{T_1 - \nu_1}{2\eta} \right) +
+ \left[ \rho_1 (\sigma^2/4) - C_1^2(\sigma^2/4) \right] \left( \frac{T_1 - \nu_1}{2\eta} \right) \left( \frac{y_1 - T_1}{2\eta} \right).
\]
The first summand is the principal’s expected payoff in the second period if first period observed output reveals the respective agent’s type multiplied by the probability that his type is revealed by first period output. The second summand is her expected payoff if first period output does not reveal additional information about the agent’s type, multiplied by the probability that this occurs.

**Proposition 1.** (The first period targets) If production is sufficiently noisy, the principal’s belief function (1) takes on three possible values and she accounts for the noise in devising the first period output targets. Specifically,

\[ 2\eta > (\theta^2 / 2) - C_1(\theta^2 / 2) \Rightarrow \]

\[ y_1 = (\theta^2 / 2) - \delta(\theta^2 / 2)(\theta^2 / 2) \gamma_1 \left( \frac{1-\theta^2}{1-p^2\theta^2} \right), \quad \text{and} \]

\[ (12) \]

\[ y_1 = C_1(\theta^2 / 2) + \delta(\theta^2 / 2)(\theta^2 / 2) \gamma_1 \left( \frac{1-\theta^2}{1-p^2\theta^2} \right)^2. \]

**Proof.** Suppose first that the principal does not take into account the effect noise has on the high type’s incentive compatibility constraint and her own future expected payoff. Then the first period target output levels are those of the static environment and hence calculated analogously to Equations (6) and (7) as \( y_1 = (\theta^2 / 2) \) and \( y_1 = C_1(\theta^2 / 2) \).

But then, given the assumption on \( \eta \) stated in the proposition, \( y_1 - \eta < \sum_1 + \eta \), so in equilibrium there exist certain output levels for which the principal cannot infer the agent’s type. This says that the principal needs to take into account the impact of noise on her future payoffs and the high type agent’s reward when designing the first period contract.

Applying the binding constraints (9) and (10), and inserting the principal’s future expected payoff (11) into the principal’s first period problem stated at the outset of this section (see Equation (8)), the first order conditions are sufficient for the solution. Simple manipulation of the first order conditions yield the first period targets stated in the Proposition.

Up to here it is demonstrated that in sufficiently noisy situations the principal will account for the noise in devising the first period targets, assuming that, in equilibrium, full separation of the types is optimal. The following Proposition demonstrates that noise actually yields that the fully separating equilibrium contract in the first period is indeed optimal.

**Proposition 2.** (Fully separating first period contract) If production is sufficiently noisy, the optimal first period contract based on observable output separates the first period targets.

**Proof.** A separating first period contract is optimal if the low type’s incentive compatibility constraint is slack, given the first period targets specified in Proposition 1. Using the fact that the low type’s individual rationality constraint is always binding, the low type’s equilibrium utility is the reservation utility of 0. Therefore the low type’s first period incentive compatibility constraint needs to assure that when targeting the high type’s level of output, the low type ends
up below his reservation utility. Hence the low type’s incentive compatibility constraint can be written as $\tau_1 - \left(\frac{v_1}{\theta}\right)^2 \leq 0$. Substituting $\tau_1$ from Equation (9), and $\zeta_1$ from Equation (10) one obtains,

$$\left(\frac{v_1}{\theta}\right)^2 + \left(\frac{y_1}{\theta}\right)^2 - \left(\frac{y_1}{\theta}\right)^2 + \delta(\theta^2/4)(1 - \Theta^2) \left(\frac{v_1 - y_1}{2\eta}\right) - \left(\frac{v_1}{\theta}\right)^2 \leq 0.$$ 

Now notice that the following inequalities are equivalent to the above condition:

$$\delta(\theta^2/4)(1 - \Theta^2) \left(\frac{v_1 - y_1}{2\eta}\right) \leq \left(\tau_1 - \frac{y_1}{\theta^2}\right)^2$$

$$\delta(\theta^2/2)^2 \left(\frac{v_1 - y_1}{2\eta}\right) \leq \left(\tau_1 - \frac{y_1}{\theta^2}\right)^2$$

$$\delta(\theta^2/2)^2 \leq (\tau_1 + y_1)^2 \eta.$$ 

Substituting $v_1$ and $y_1$ from Proposition 1 into the right-hand-side of the inequality, one obtains

$$\delta(\theta^2/2)^2 \leq \left(\frac{v_1}{\theta^2} + C_1(\theta^2/2)\right) 2\eta + \Delta,$$

where $\Delta$ does not depend on $\eta$.

Clearly the right-hand-side is increasing in $\eta$, whereas the left-hand-side is not, so for sufficiently large $\eta$ the first period contract is separating. $\Box$

Underlying this result is the impact of noise on the additional up-front payment made to the high type. Recall that the additional up-front payment is equal to the good agent’s discounted gain from deception. Unlike in the deterministic setting, the high type does not lose all expected informational rents even when choosing the equilibrium action, nor does he gain as much by mimicking the low type, since deception need not be successful. Indeed, the gains from deception diminish as noise increases, so that the additional reward paid to the high type is no longer large enough for the low type to be tempted by the take-the-money-and-run strategy.

The optimal output targets as well as the expected rewards implied by the binding constraints (9) and (10) are depicted by the points $\bar{1}$ and $\bar{1}$ in the diagram, where the feasibility of separation is reflected by the fact that the high agent’s output target and expected reward (point $\bar{1}$) lies below the low agent’s reservation level indifference curve (denoted by $\mu = 0$). Once again, the low agent’s target and expected reward pair (point $\bar{1}$) lie on his reservation level indifference curve — reflecting the binding individual rationality constraint. The dashed line in the diagram represents a first period indifference curve of the high type (as derived in Section 5). The fact that both $\bar{1}$ and $\bar{1}$ lie on this curve reflects the binding incentive compatibility constraint.

Thus far we have assumed that the contract between the principal and the agent is based solely on observed output. However, given the standard results from deterministic contracting environments, Lemma 1 in conjunction with
Propositions 1 and 2 serve as a basis to demonstrate that a larger contract space, e.g., one in which contracts can be based on additional messages from the agent to the principal, does not yield a superior contract. Formally,

**Corollary 1.** (Optimality of output–based contracts) *If production is sufficiently noisy, even if the agent can send an additional message to the principal to reveal his type, and this message can be contracted on, the contract based exclusively on observable output remains optimal.*

**Proof.** Although one can formally derive the best possible contract that the principal could devise using the enlarged message space in a non-trivial way and compare it to the contract based on Proposition 1 to prove the Corollary, there is a straightforward intuitive argument that makes the same point. It relies on the simple observation that when the principal cannot commit to using information gleaned about the agent’s type against the agent in future interactions, information revelation can be damaging to the principal’s interests. Put another way, provided that no additional messages are sent to the principal, the noise serves as a commitment device for the principal.

Formally, consider a contract that provides that the agent (truthfully) announces his type to the principal. As a consequence of the risk–neutrality of the players, contracts would have identical target outputs to those in the deterministic case. As is well–known for the deterministic case, a contract with long–term commitment that simply repeats the short–term asymmetric information contract is superior for the principal compared to the contracts that arise without long–term commitment.

Now notice that the following argument holds for sufficiently noisy production provided the principal does not allow messages beyond observable output to be sent. Proposition 2 shows that the targets are given in Proposition 1. With enough noise, these targets are arbitrarily close to the static asymmetric information outputs, given in Equations (6) and (7). By Lemma 1 the probability that the principal is able to infer the agent’s type is then arbitrarily close to zero, so that the second period contract again specifies expected targets arbitrarily close to the static asymmetric information outputs.

Thus, provided no additional messages are used, sufficient noise in production makes the sequence of contracts arbitrarily close to the repeated static asymmetric contract, which is better than the principal can do without long–term commitment in the deterministic case, or, equivalently, when using additional messages. \(\Box\)

The intuition is obvious: The principal maximizes her payoff, not her knowledge. When obtaining information is costly due to the fact that the principal cannot commit to not using the information obtained to exploit the agent, then having less than full information may be better than full revelation.

It is worth noting that while the proof relies on a limiting argument, the actual critical level of noise need not be particularly large at all.

Nevertheless, the principal accounts for noise in production and the dissemination of information when determining optimal targets. In fact, the principal
manipulates the flow of information to increase the principal’s expected payoff. This critical difference when compared to a deterministic setting is studied in detail in the next section.

4 Experimentation and signal dampening

Having derived the first and second period equilibrium output targets, consider now the dynamics of the contract from the first to the second period. This allows an analysis of the ‘ratchet effect.’

Since the ratchet effect was introduced into the literature the term has been used with more general meaning. Namely, the phenomenon that in dynamic asymmetric information hidden action settings, ‘high type’ agents who stand to lose informational rents if information dissemination takes place, have a strong incentive to keep their information private. Indeed, a feature of modern contract theory, as studied in deterministic settings, shows that when the first period contract is fully separating, the ‘ratchet’ takes hold of payments made to the high type agent, not, however, his targets. That is, the targeted output remains fixed for the high type, while his reward is ‘ratcheted’ downward as the principal observes the agent’s type. Interestingly, a ratcheting of targets does take place in deterministic models, however, in the opposite way of the original meaning. That is, a low type agent will have his target increased as his type is revealed.

Nevertheless, the ratchet effect is still commonly understood to apply to a superior agent’s targets over time. Specifically, one commonly sees expectations regarding the performance of good agents increased, as opposed to payments made to good agents cut.

In a stochastic setting both first period targets are used to manipulate the flow of information in the course of the interaction between the principal and the agent. Therefore, neither type of agent has his target remain fixed over time. In particular, due to the impact of noise on the two contracts, the conventional ratchet effect is observed in equilibrium. That is, good agents indeed have their targets tightened in the course of the interaction, so that they are expected to exceed previous performances.

**Proposition 3.** (The Ratchet effect) If production is sufficiently noisy, the ratchet effect occurs. That is, the high type agent has his target output level increased from the first to the second period. Specifically,

\[ \bar{y}_1 < \bar{y}_2. \]

**Proof.** A simple comparison of the targets yields the result. Specifically, the first period target of the high type is given by Equation (12) in Proposition 1. Whereas the high type’s second period target is given in Equation (6). A comparison yields

\[ \bar{y}_1 = \bar{y}_2 - \delta \theta_2 (1/2) \theta_2 (1/2) \rho_1 (1 - \Theta^2)^2 / 4 \eta_1 - \rho_1 \Theta^2. \]
Thus, $y_1 < y_2$. □

Although, given the two equilibrium contracts, the proof is straightforward, underlying the result are rather complex informational issues that warrant closer examination. In particular, the source of the discrepancy between the two targets stems from the role of slow information transmission in stochastic environments and the fact that the principal affects the flow of information when devising the equilibrium targets. Indeed, two countervailing forces lead to the result. On the one hand the principal values information and would like to become fully informed about the true state of the world before designing the second period contract. This would enable her to devise the ‘first–best’ contract for the given state of the world and thus extract all informational rents from the agent.

On the other hand, the principal cannot commit to not extracting all informational rents if indeed she does become informed about the true state of the world. Consequently the principal must make a substantial up–front payment to the high type in order to prevent him from deceiving the principal by mimicking the low type. Since this up–front payment is costly to the principal, she has an incentive to keep it small.

Both of these forces depend on the degree of information transmission due to observing first period output. However, in specifying the first period target levels, the principal has control over the flow of information in equilibrium. This becomes clear when one considers the principal’s updated beliefs given in Lemma 1. The belief function depends on the equilibrium targets of the first period.

Thus, in regard to her incentive to learn the true state of the world, the principal can choose first period targets of the two types far apart to make the signal more informative. This is done by increasing the distance between the two first period targets in order to increase the probability that the principal infers the agent’s type, i.e., by increasing $\left(\frac{y_1 - y_2}{y_1}ight)$. The impact of this incentive on the agent’s first period target is demonstrated in the following Theorem.

**Theorem 1.** (Experimentation) In determining the optimal first period targeted signal, the principal experiments. That is, the principal’s future payoffs, $E v_2$, are increasing in the distance between $y_1$ and $y_2$.

**Proof.** The principal’s future expected payoff is given in Equation (11). Thus,

$$\frac{\partial}{\partial (y_1 - y_2)} E v_2 = \frac{\rho_1 C_1 (\theta^2/4)(1 - \Theta^2) + (1 - \rho_1)(\theta^2/4)}{2\eta} - \frac{(1 - \rho_1) C_1 (\theta^2/2)(1 - (C_1/2))}{2\eta} > 0,$$

since $\left(\frac{\theta^2}{4}\right) > C_1 (\theta^2/2)(1 - (C_1/2))$. □

Consider now the up–front payment made to the high type in the first period in order to allow for learning. Given the environment, the principal is unable
to commit to a long-term contract, and thus not able to commit to not using the information gleaned from first period output to extract informational rents from the high type in the second period. However, just as the principal can manipulate the flow of information to experiment and learn more, the principal can manipulate the flow of information in order to preserve the high type’s future informational rents. This also reduces the agent’s gain from attempted deception. Consequently, the amount of the up-front payment necessary to induce him to target the equilibrium output of the first period is reduced. Indeed, the closer the first period target levels of output are, the smaller is the up-front payment the principal must make to the high type. In regard to the high type’s first period target the result of this incentive is given in the following theorem.

**Theorem 2.** *(Signal dampening)* In determining the first period optimal signal, the principal chooses to preserve informational rents of the high type in order to reduce the up-front payment to the high type. That is, the up-front payment to the high type is increasing in the distance of the first period output targets.

**Proof.** Given the up-front payment from the high type’s first period incentive compatibility constraint (see 9),

$$\frac{\partial}{\partial (y_1 - y_1)} \tau_1 = 2 \frac{y_1 + y_1}{\theta} + \frac{\delta (\theta^2 / 4) (1 - \theta^2)}{2 \eta} > 0,$$

where the first term in the derivative is the same as in the static problem and the second is the impact of signal dampening.

The principal’s benefit from reducing the up-front payment to the high type exceeds the gains from experimentation, so that the net effect is a reduction in the high type’s first period target and an increase in the target levels in the course of the interaction — hence the ratchet effect.

5 The equilibrium reward schedules

After having determined the target output levels and implied rewards for the two period interaction in the previous sections, consider now the structure of the optimal contracts that ensure that the two types of agents target the output levels intended for them.

The contract offered in any given period is a reward schedule mapping observed levels of output into rewards paid to the agent by the principal. Thus, let \( r_i(y_t) : \mathbb{R} \rightarrow \mathbb{R} \) denote the equilibrium contract in the \( t \)-th period. The equilibrium reward schedule must fulfill two conditions. First, it must be assured that an agent who chooses the equilibrium effort level receives the reward implied by the equilibrium targets and the binding constraints. Second, it must be the case that neither type of agent can increase his utility by choosing any other than the equilibrium target designed for his type.

To understand the agent’s incentives for targeting any particular expected output level, it is best to analyze the agent’s indifference curves over targeted
outputs and rewards. For a static game, the indifference curves are simply implied by the agent’s instantaneous utility given by Equation (2). For both agents the second period is essentially static. Moreover, since the low type’s first period actions do not affect his second period payoff, the low type’s first period indifference curves coincide with his second period indifference curves.

Consider now the high type in the first period. Recall that the high type’s second period expected payoff depends on the principal’s beliefs, which, in turn, depend of the first period observed output. Thus, the high type’s total utility, that is his expected utility for the entire two-period game, depends on the first period effort level in two ways. It impacts the utility of the first period due to the effort and anticipated first period reward, and that of the second period through the impact of the observed output on the principal’s beliefs and thus the second period contract. One thus needs to consider the impact of the high type’s first period effort level on the second period payoff when determining the high type’s first period indifference curves. Let $u^{(2)}(E_{Y1}, r_1)$ denote the high type’s total expected utility for the game as a function of a first period targeted output $E_{Y1} = \theta e_1$, given Equation (1), and first period reward of $r_1$.

**Lemma 2.** The high type’s total expected utility as a function of his first period targeted output and reward is,

$$u^{(2)}(E_{Y1}, r_1) = r_1 - \left( \frac{E_{Y1}}{\theta} \right)^2 + \delta \frac{\eta}{4} \left( 1 - \Theta^2 \right) \left( \frac{\tau_1 - E_{Y1}}{2\eta} \right) + \delta C_1^2 \left( \frac{\theta^2}{4} (1 - \Theta^2) \left( \frac{\tau_1 - \tau_e - 2\eta}{2\eta} \right) \right).$$

Proof. The first part of the expression is the high type’s expected utility in the first period, i.e. the payment minus his disutility from work. The second part is the agent’s discounted second period payoff if the principal thinks that the agent is a low type, multiplied by the probability that the principal has these beliefs given the agent’s first period effort (in equilibrium this probability is 0, because $E_{Y1} = \tau_1$). The third part is the high type’s discounted utility in the second period if the principal’s posterior coincides with the prior, multiplied by the probability that this occurs.

The implied indifference curve is given by the dashed line in the diagram ($\pi^{(2)} = \text{const.}$). It is steeper than the static indifference curves of the high type ($\pi = 0$ and $\pi = \text{const.}$), because each increase in first period effort brings about a decrease in future expected rents for which the agent must be compensated for in the present. For now, the relevant observation about the implied indifference curve is that it is convex in the agent’s targeted output, $E_{Y1}$. Keeping this in mind, we turn to the equilibrium contract.

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These are represented by the solid curved lines in the diagram.
Much research has focused on optimal reward functions for moral hazard situations in principal agent models (see, e.g., Hart and Holmström, 1987). A common problem is that when agents are risk averse optimal reward functions have highly unusual shapes, and such optimal functions are usually not found in the real world. Holmström and Milgrom (1987) derive conditions under which linear schemes are optimal. Although the agent is risk neutral, a linear scheme cannot be found to support the equilibrium implied by Sections 3.1 and 3.2, due to the convexity of the high type’s indifference curves. However, another common reward structure found in the real world is that of a multi-tier reward, in which a base salary is paid and additional bonuses are added to the base salary upon observation of particularly high output levels. We restrict attention to the simplest of such structures, namely a two-tier reward function, in which for all equilibrium output levels the agents receive a base pay, and for output observations above a certain critical level, $y_c$, a fixed bonus is added to this pay.

Since the problem is quite similar in both periods, we focus only on the first period contract, analogous results apply to the second period. Moreover, for simplicity we specify the reward function only for equilibrium observations.

** Proposition 4.** (First period contract) There exists $y_c \in [y_1 - \eta, y_1 + \eta]$ such that

$$r_1(y_1) = \begin{cases} (\frac{y}{\theta})^2, & \text{if } y_1 \in [y_1 - \eta, y_c), \\ (\frac{y}{\theta})^2 + \left[\frac{y_1 - y}{\theta^2} + \delta \frac{y_2}{\theta} (1 - \Theta^2) \frac{y_1 - y_c}{2\eta} \right] \frac{2\eta}{y_1 - y_c}, & \text{if } y_1 \in [y_c, y_1 + \eta]. \end{cases}$$

constitutes an equilibrium contract.

**Proof.** Notice first that if a low type chooses the equilibrium target $y_1$, his expected reward is $(\frac{y_1}{\theta})^2$, since $y_c > y_1 + \eta$. If the high type chooses the equilibrium target $\overline{y}_1$, he receives $(\frac{y_1}{\theta})^2$ with certainty, and the added bonus with probability $\frac{y_1 - y_c}{2\eta}$. Thus, in equilibrium, both agents obtain the rewards implied by the binding constraints (9) and (10).

Given the distribution of noise, and the reward function, the expected reward as a function of the targeted signal is convex and piecewise linear. Specifically, it is a constant so long as $Ey_1 + \eta < y_c$, thereafter ascends linearly. Thus it is clear that for sufficiently large $y_c (\leq \overline{y}_1 + \eta)$ the expected reward lies below the high type’s indifference curve everywhere with the exception of $y_1$ and $\overline{y}_1$. \(\square\)

The expected reward function (i.e., $Er_1(Ey_1)$) is depicted as the thick piecewise linear function connecting the points $\underline{y}_1$ and $\overline{y}_1$ in the diagram. Beyond this interval, entails the possibility of out–of–equilibrium output observations, which can easily be ruled out. The second period contract follows *mutatis mutandis* if the agent’s type has not been gleaned, otherwise a straightforward full information forcing contract applies.
6 Conclusion and outlook

In this paper we study a two period principal agent relationship, in which the true state of the world can take on two possible values and is “time-invariant.” We assume that contracts are short term, so that after the first period the principal designs a second contract for the second period, taking the information available at that stage of the game into account — the classic environment in which the ratchet effect is of concern. These settings are well-understood in deterministic environments, that is, instances in which actions and outcomes are deterministically linked. We study this setting when the link between an action and the outcome is stochastic, that is, affected by noise. Two fundamental differences between deterministic settings and stochastic ones emerge.

First, the rewards necessary to induce “truth-telling” are lower when the environment is stochastic. To obtain a separating equilibrium, agents, to whom informational rents accrue due to the asymmetric information, must be paid a reward to reveal their type. In a stochastic environment this reward is lower because, with incomplete learning, there is a positive probability that even when choosing the “proper” action, the principal may still not be able to infer the true state of the world. In this event the agent’s informational rents continue to accrue in future periods. In addition, noisy production makes deception on the part of such an agent less effective, because even if the agent tries to deceive the principal, there is a positive probability that, due to noise, the deception is not complete. As a consequence of this, a ‘fully separating’ contract is optimal in the first period.

The second effect of noise impacts the choice of the first period actions specified by the principal in the first contract. Two considerations play a part in determining the equilibrium actions. These considerations work in opposite directions. First, by the choice of the actions specified in the first period contract, the principal is able to increase the effect noise has on the rewards. Indeed, the principal has an incentive to choose the equilibrium actions in such a way as to reduce an agent’s gain from deceiving the principal. This is done by making the signal that the principal observes less informative, thus protecting some of the agent’s rents in the second period. The amount of second period rent that thus accrues to the agent need not be paid as an additional reward in the first period. In other words, although the principal cannot commit to a long term contract under which she does not exploit the information gleaned, she can affect the degree of information transmission by choice of the first period actions.

The second impact of noise on the equilibrium targets stems from the principal’s desire to experiment. Since information regarding the true state of the world increases the principal’s payoffs in the second period, the principal increases the informativeness of the first period signal. In other words, since information is valuable to the principal in the second period, the initial contract is used as a mechanism for the principal to increase the amount of information.

As a result of the impact of noise on the equilibrium targets, the original ratchet effect is observed. That is, unlike the deterministic setting in which the
high type’s target is fixed over time, and the low type’s target is increased, in a stochastic setting, the high type’s target is ratcheted up in the course of the interaction, so that good past performances lead to increased expectations.

References


