Favouritism and cartel disruption in first-price auctions

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Abstract

The seller in an auction will generally not be happy to know that a cartel of bidders will take part in that auction. Cartels generate their profits by inducing a final price which is lower (higher in the case of procurement contracts’ auctions) than in a competitive auction. This paper proposes a solution to the problem. By allowing the seller to cheat on the auction rules, and to allocate the good to a given bidder with a predetermined probability (favouritism), we show that when no cartel is active, the auction leads to a lower price than that obtained in a purely competitive auction. However, if a cartel is operative, favouritism generates incentives for the favoured bidder to defect the cartel. This single defection is sufficient to disrupt the cartel. In equilibrium, the seller may choose this probability of cheating so as to obtain the highest possible final auction price, which we show to be a second-best outcome. In other words, this proposed solution to the cartel’s existence does not lead to a final auction price as high as that obtained in a competitive auction.

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1 Introduction

In an auction, when it becomes known that a cartel of bidders has been formed, the seller should not be happy about it. If the seller is the government (or the buyer in the case of procurement contracts) and there is only one cartel (which includes a significant number of bidders), that may make things worse\(^1,2\). That may be the case because (i) in procurement contracts there are usually fewer bidders (and therefore already limited competition in the auction) and (ii) because of the public profile of such auctions (which makes the government directly accountable for the auction outcome). A cartel acts as a monopolist, with its own bargaining power over the final price of the good for sale, which means that the seller will not end up with the profits it would obtain in a competitive auction. Needless to say, any kind of cartel behaviour is illegal, but can only be punished if it is discovered.

First-price auctions are particularly popular in procurement contracts\(^3\). In some markets\(^4\), bidders may be dependent on winning some auctions so that there is a strong incentive to cooperate and form a cartel. This incentive may not be so much related to the potential threat of bankruptcy if they do not win, but more with the possible spillover effects of government auctions: a bidder may be willing to win the government auction at a lower price if that allows him to increase the probability of winning other public or private contracts.

McAfee and McMillan (1992) analyse in depth a cartel’s optimal strategy (in an independent private values model). In their paper, they assume a cartel may be weak or strong: a weak cartel cannot make transfer payments among its members, whereas a strong cartel can. Strong cartels are easier to detect, because they necessarily involve more communication among members; however, McAfee and McMillan (1992) have shown that strong cartels are more profitable than weak cartels\(^5\). This form of explicit collusion is illegal in many countries, and provided the punishment is sufficiently large, strong cartels are hard to sustain. Weak cartels are harder to detect, because the type of collusion they involve may be obtained even without explicit communication, as McAfee

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\(^1\)In the United States, the Federal Government sells (by means of an auction) a substantial number of commodities or rights, such as Treasury notes and bills, timber rights and offshore oil leases (Graham and Marshall (1987)). Many European countries follow similar procedures.

\(^2\)In first-price sealed bid procurement auctions, the government is the buyer, and the auction winner is whoever submits the lowest bid. Throughout this paper, we assume that there is one seller and several bidders, and that the winner is whoever submits the highest bid.

\(^3\)See Klemperer (1999).

\(^4\)For example, in the USA, the defence industry is critically dependent on the number and size of contracts signed with the Pentagon. In developing countries, such as Portugal or Spain, the construction sector is highly dependent on the construction contracts auctioned off by the government.

\(^5\)McAfee and McMillan (1992) define the cartel’s profits as the sum of each individual member’s profits.
and McMillan (1992) have shown.

Collusion is illegal in the United States, as in many other developed countries⁶. However, a weak cartel can use an implicit bidding strategy which, although optimal to all cartel members⁷, does not require communication between its members or even transfer payments. This implicit bidding strategy is to have all cartel members bid the reserve price, in which case the seller must allocate the contract or item for sale by randomising over all bidders who submitted bids. McAfee and McMillan (1992) prove that, for most signal distributions, bidders prefer this implicit agreement to a competitive auction, because it yields higher expected profits. In the real world, according to Scherer (1970), “Each year the federal and state governments receive thousands of sets of identical bids in the sealed bid competitions they sponsor [...]”⁸. And although this fact clearly hints at some sort of collusive agreement between the bidders, lawyers in the United States are “[...] still discussing the nature of the evidence that is needed in order to prosecute for collusion.”⁹.

McAfee and McMillan (1992) also show that a strong cartel, which can make transfer payments among its members, is more profitable than a weak cartel, although it faces a higher risk of detection and subsequent prosecution. In this case, the optimal cartel mechanism is to hold its own (illegal) auction. The winner in the cartel auction will be the only bidder to make a bid in the real auction, and he will bid the reserve price. The difference between the reserve price and the bid he made in the cartel auction is then distributed among all cartel members, who receive strictly higher profits than in a weak cartel. Not only is this mechanism optimal, but it is also efficient: the bidder who values the item the most will win the auction¹⁰.

Feinstein, Block and Nold (1985) also analyse cartel formation and behaviour in a multiperiod auction, in which the cartel can successfully manipulate information so as not to be detected and maximise its profits over time.¹¹ As in the model of McAfee and McMillan (1992), cartels may succeed in extracting the seller’s profits, and still go undetected.

Feinstein et al (1985), McAfee and McMillan (1992) and Hendricks and Porter (1989) have conjectured that an unannounced change of rules (which we define as cheating) may be successful at disrupting a cartel. There are obviously many different ways of cheating: the seller can select a winning bid which was not the highest, he can refuse to allocate the good if he suspects of collusion,

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⁶E.g. all countries within the European Union.
¹⁰Whereas with a weak cartel, the auction may not be efficient: there is a positive probability that the bidder who values the item the most does not win the auction.
¹¹LaCasse (1995) also analyses collusion in first-price sealed bid auctions.
he may change the reserve price and re-auction the good, etc. In their paper, McAfee and McMillan (1992) propose three possible responses by the seller when he knows that a cartel is operating. He can raise the reserve price, and hence force a (weak or strong) cartel to make a higher bid; he can keep the reserve price secret, as in many English auctions\textsuperscript{12}, and thus force the cartel members to communicate in order to agree on which bid to make (and communication increases the probability of detection); or the seller should make the environment as stochastic as possible\textsuperscript{13}, thus increasing the probability that the cartel will use trigger strategies on its members to enforce the collusive mechanism\textsuperscript{14}. McAfee and McMillan (1992) conclude their paper suggesting that “[...] as a general rule, the seller should make the environment as stochastic as possible [...]” as a way of preventing a cartel from successfully rigging the bids.

This paper proposes a method of introducing a stochastic component in the cartelised bidding environment. According to the rules of a first-price auction, the winner must be the bidder who has submitted the highest bid. In our model, we allow for the possibility of the seller cheating and not following those rules. i.e. in our model, with some (positive) probability, the winner will not necessarily be the bidder who has submitted the highest bid. Additionally, we allow for the seller being able to choose the auction winner when he does cheat. He may randomly pick one bidder (with equal probability) from the set of submitted bids or he may assign different probabilities of winning to each of the submitted bids. In our definition, favouritism occurs when the seller cheats and chooses one particular bidder with probability one (who we define to be the favourite bidder). Also, our model concentrates on weak cartels. Strong cartels can easily be ruled out once the punishment for detection is sufficiently large. Weak cartels, on the other hand, can have equally likely pervasive effects without risking detection. Hence, we believe they should be the main concern of the seller.

When the good for sale is a governmental contract, bids are usually in the form of cost estimates and other more subjective factors, such as time needed to complete the task, estimates of any additional costs, financial health of the company, etc.\textsuperscript{15} These other subjective factors are ideal as a means of justifying the highest bid not winning the auction. These factors may be used to favour one particular bidder (or bidders) in a sealed-bid auction, without risking legal action from

\textsuperscript{12}See Cassady (1967).
\textsuperscript{13}For example, occasionally choosing the wrong winning bidder (provided the winner’s bid is not revealed).
\textsuperscript{14}And thus induce more competitive behaviour from all bidders as a punishment to the deviating bidder.
\textsuperscript{15}In procurement auctions, the government is the buyer and the lowest bid will win the auction. Without loss of generality, we assume here that the government is actually the seller, and therefore that the highest bid will win the auction.
the “legitimate” winner (the bidder who submitted the highest bid).

Whereas, at first sight, this may seem motivated by vested interests\textsuperscript{16}, we will show that this may be a way of preventing cartel formation. By favouring one particular bidder, the seller is implicitly increasing the probability of that bidder winning the auction. Roughly speaking, provided this probability is higher than the probability of winning in a cartel, this (favourite) bidder will prefer to defect from the cartel. If such a defection occurs, no other bidder will benefit from bidding as a cartel: they will all prefer to bid competitively (with the exception of the favourite bidder)\textsuperscript{17}. Thus, the subjective factors we have just mentioned may be seen as the “public” justification of this anti-cartel mechanism.

Such behaviour is obviously illegal. In most first-price auctions, the winner must be the bidder who has submitted the highest bid. However, in many if not most government auctions, a bid is not simply the price. In motorway contracts, for example, the government is also interested in the time it takes to build the motorway, in its quality, in the financial health of the bidder and his ability to carry out his contractual obligations. A bid, in these cases, is typically multidimensional. In some government auctions (e.g. the auction for a digital terrestrial TV channel in Portugal\textsuperscript{18}), the winner may not be the bidder who proposed to pay more for the good for sale. In a standard first-price auction with unidimensional bids, this would be illegal. The price would be the only dimension to be evaluated. But in the case of multidimensional bids, the government can always argue that it values other factors more than the price. Appeals of decisions in government auctions are not uncommon, but, in general, prosecuting the government is an expensive and time consuming task for a losing bidder. Moreover, because the choice between multidimensional bids is necessarily subjective, the probability of winning such appeals is necessarily quite small. Additionally, in many auctions, the bids are not always revealed to the public (including all the bidders). Hence, the winning bidder may never know whether his bid was indeed the highest bid.

We argue that these multidimensional bids provide the ideal mechanism for the seller to cheat and choose a bidder who it would rather see winning the auction (favouritism) instead of the bidder

\textsuperscript{16}In the case where the government owns shares of a company, awarding the contract to it will increase the value of the government’s shares.
\textsuperscript{17}We will prove this result in Section 5.2.
\textsuperscript{18}In July 2001, the Portuguese Institute of Communications (the telecoms regulator) auctioned off one digital terrestrial television licence. The winning consortium (which includes RTP, the government-funded TV channel, and SIC, a privately owned TV channel) proposed to charge approximately 117 euros for the set-top box and then a monthly fee of approximately 12 euros whereas the losing bidder (a consortium formed by Oni, a subsidiary of EDP, the main electricity supplier, and TVI, a private TV channel) would charge 24 euros for the set-top box and a monthly fee of 7 euros. Therefore, price-wise, the winning bid was not the best bid. The losing consortium decided to appeal the regulator's decision (in \textit{Expresso}, 21/7/2001: http://online.expresso.pt/economia/artigos/interior.asp?cdicao=1499&id_artigo=ES31692).
who proposed to pay the highest price. With multidimensional bids, the seller can get away with it because it will be hard to prove that the winning (multidimensional) bid was not the best. With such multidimensional bids, the winning bid is usually the best weighted average of all the factors contained in it, and these weights are necessarily subjective. In auctions where the bids are not revealed ex post, bidders can never know whether the seller has cheated, and this is ideal for favouritism to occur, without the threat of future prosecution by any of the bidders.

McAfee and McMillan (1992), Feinstein et al (1985) and Hendricks and Porter (1989) suggest that active seller behaviour may disrupt any existing cartel, giving incentives to each individual bidder to bid competitively in the auction. We prove this result in our model. By selecting the probabilities of (i) allocating the good to a particular bidder when he decides to cheat, as well as (ii) the overall probability of cheating, the seller can generate incentives for a bidder to defect the cartel.

We show that there exists an optimal choice of the probability of cheating and of the probability of any given bidder being awarded the good when the seller does cheat. The optimal choice of these probabilities is to have the seller choose the lowest possible probability of cheating (which still generates cartel disruption) and select a particular bidder who will be the winner with probability one when he does cheat (favouritism). Such behaviour induces all bidders (except the favourite bidder) to bid competitively (as they would in a standard first-price auction), whereas the favourite bidder will submit a lower bid than in a competitive auction (because his probability of winning is higher than anyone else’s - he wins with probability one when the seller cheats, and therefore does not need to bid as aggressively as he would in a competitive auction).

Such a strategy is optimal because it generates the highest possible expected price. In our model, setting a high probability of cheating will induce all bidders to lower their bids (from the purely competitive auction optimal bid); by contrast, setting a low probability of cheating and a high probability of selecting a particular bidder (favouritism) when the seller cheats will induce this particular bidder to lower his bid, but all other bidders will bid as in a purely competitive auction.

In this context, the expected winning bid is more likely to come from the set of bids submitted by

\footnote{In our model, the seller’s probability of cheating is made public before the auction starts; an auction model where the bids are not revealed after the auction has finished must account for the bidder’s beliefs of the seller’s probability of cheating. The problem with such a model is that a weak cartel (which requires very little communication between members) would break down, because each individual member would benefit from deviating from the cartel’s prescribed (minimum) bid. The rationale is that such deviations would not be detected. Hence, such a model would have to impose stricter incentive compatibility constraints on the cartel members (e.g. becoming a strong cartel).

\footnote{We will later define a purely competitive auction as an auction in which the seller never cheats (i.e. the standard first-price auction model).}
all bidders other than the favourite (i.e., all the bidders who submit purely competitive bids), and hence the expected price is maximised.

In some government auctions\footnote{E.g. the auction of the 4th TV channel in Portugal in the early 1990’s, or some motorway contracts, where private companies compete with publicly funded companies.}, the decision to allocate the item to a particular bidder is often contested in court. Other bidder’s grounds for appeal are claims that the winning bid was not the best. Although political reasons\footnote{Lobbying is a successful tool at the disposal of the defence industry in the United States; in auctions where public sector companies win contracts, instead of the private companies, vested interests may be the implicit reason for favouritism: by awarding valuable contracts to its own companies, the government is inflating its value, and hence the profits it receives as a shareholder.} may be seen as the most intuitive and obvious explanation of favouritism, the model in this paper shows that appearances may be deceptive: favouritism is effective in preventing cartel formation. It is difficult to draw a line to separate auctions in which favouritism is used as an anti-cartel mechanism and auctions in which it is simply driven by corruption or lobbying. The most important contribution of this paper is to show just how thin that line can be; in fact, from the bidder’s point of view, it is never clear when cheating is used as an anti-cartel device or when it is simply the result of corruption.

The theoretical model which allows for cheating is presented in the next section. Section 3 analyses the optimal bidding behaviour in a competitive auction (when the seller is allowed to cheat) whereas Section 4 discusses optimal cartel behaviour in such a model. Section 5 shows how cheating and favouritism are successful tools to disrupt a cartel and Section 6 concludes.

2 The model

Each bidder $i$ has a private valuation of the good for sale, known only to himself, which we denote by $v_i$, for $i = 1, \ldots, n$. We assume these to be independent across bidders, and coming from a common distribution, $f(v)$, with support $[0, 7]$. There exists a corresponding cumulative function which we denote by $F(v)$. This independent private values model may be seen as restrictive, because it does not encompass the sale of goods whose value has an unknown common component (see Milgrom and Weber (1982) for a general model), like oil tracts or pieces of art. When auctioning art, the perceived value of a painting is usually a mixture of how much one likes the painting (the private value component) and how much one believes it will be worth in the future (the common value component). However, the independent private values model is still applicable in situations where all bidders have the same information on the unknown common value component. For example, if each bidder’s total valuation is $w_i = z + v_i$, where $z$ is the common component of the good (the
same to all bidders) and \( v_i \) is each bidder’s personal valuation (as previously explained), and \( z \) is independent of \( v_i \) for every \( i \), then the independent private values model still holds. It is only when the value of \( z \) varies across bidders (or bidders perceive it differently) that this model is no longer valid. We also assume the seller sets a reserve price of \( 0.23 \)

Let us define \( \lambda \) as the probability of cheating by the seller. Thus, with probability \( \lambda \), the seller will not follow the standard auction rules, and will choose the winning bidder in some other way (which we will shortly define); with probability \( (1 - \lambda) \) he will not cheat, and will sell the good to the highest bidder. In an auction model with \( n \) bidders, the seller’s expected profit is given by:

\[
E [\pi^S] = E [\lambda (b_i) + (1 - \lambda) (\max (b^*))]
\]

(1)

where \( b^* = (b^*_1, \ldots, b^*_n) \) is the vector of submitted bids and \( b_i, i \in (1, \ldots, n) \) is the winning bid when the seller cheats (with probability \( \lambda \))^24. When the seller cheats, we will assume he assigns a probability of winning to each bidder, which we define as \( \theta_i = \Pr [i = \hat{i}] \). Hence, with probability \( \lambda \theta_i \), bidder \( i \) will be chosen as the winning bidder regardless of his bid\(^{25} \). We assume that \( \lambda \) and \( \theta_i \) are publicly announced before the auction, and hence all bidders know what their probability of winning is when the seller cheats as well as the seller’s probability of cheating. We also assume

\[
\sum_{i=1}^{n} \theta_i = 1, \text{ for all the } n \text{ bidders.}
\]

Our model is based on the assumption that by choosing a particular favourite bidder with some probability, the seller implicitly signs a contract with that bidder. Defection of the cartel is a condition of that contract, i.e. the seller favours that bidder with some probability and in exchange that bidder is required to submit a bid outside the cartel. Therefore, the favourite bidder must decide between accepting or not accepting that contract. If such a condition was not stipulated, then the favourite bidder would simply accept the seller’s improved terms (an increased probability of winning the auction) and bid within the cartel (as we will see in Section 5). Therefore, the seller’s optimal strategy is to design a contract in such a way that the favourite bidder prefers to bid outside the cartel than to bid within it, i.e. to design a contract which gives the favourite bidder

\(^{23}\)McAfee and McMillan’s (1992) model assumes a reserve price \( r > 0 \). In our model, a reserve price introduces a credibility problem on the seller’s optimal strategy: he may choose to favour a particular bidder with probability 1 when he does cheat, but then realize that this favoured bidder had a valuation below the reserve price (and who, hence, would not be interested in winning the auction when the seller did cheat, as it would leave him with negative profits). Therefore, we assume the seller will always sell the good as long as at least one bidder submits a bid larger or equal to 0. In a model of repeated (first-price) auctions, Skrzypacz and Hopenhayn (1999) make the same assumption.\(^{24}\)In which case \( \hat{i} \) is the winning bidder.\(^{25}\)The reserve price in the auction is 0. We assume that if a bidder is indifferent between bidding and not bidding (i.e. when \( v_i - 0 \)), he will always bid.
a higher payoff by bidding outside the cartel than bidding within it. As we will see, a defection from the cartel implies that the favourite bidder must submit a bid higher than the cartel’s suggested bid (the reserve price). Hence, in order to persuade the favourite bidder of submitting such a bid, the seller must increase the probability of that bidder winning; this is done by favouring him (i.e. allocating a higher individual probability of winning) when he decides to cheat.

Let us turn our attention to the bidding behaviour in such an auction. Each bidder’s expected profit, given his private valuation \( v_i \), is given by:

\[
\pi_i = \lambda (v_i - b_i) \mathbf{1}_{\{i \neq \hat{i}\}} + (1 - \lambda) (v_i - b_i) h_i(b_i) \tag{2}
\]

where \( \mathbf{1} \) is an indicator function, \( b_i \) is the bid submitted by bidder \( i \) and \( h_i(b_i) \) is the probability of winning the auction if the seller does not cheat (and the auction is competitive). We can rewrite the equation above as:

\[
\pi_i = \lambda \theta_i (v_i - b_i) + (1 - \lambda) (v_i - b_i) h_i(b_i) \tag{3}
\]

and this is the objective function of each individual bidder, which he will maximise with respect to \( b_i \). In a symmetric equilibrium, the equilibrium bid function is symmetric \( b_i(\cdot) = b(\cdot) \); it is also increasing. Hence, the probability of a particular bidder winning the auction (the probability of \( b(v_i) > b(v_{-i}) \), where \( -i \) stands for any other bidder except \( i \)) is equal to the probability of his holding the highest valuation (the probability of \( v_i > v_{-i} \)). The reserve price is 0; therefore, the probability of winning when the auction is competitive (and the seller does not cheat) is given by\(^{26}\):

\[
h_i(b_i(v_i)) = F'(v_i)^{n-1} \tag{4}
\]

The timing of events is as follows:

1. Firstly, the seller announces \( \lambda \) and \( \theta_i \), \( \forall i \). These become common knowledge.

2. Secondly, given the seller’s announcement, bidders must decide whether or not to collude and form a cartel. This decision is made before the valuations are drawn.

\(^{26}\)This is a private values model; hence, each bidder’s valuation is drawn independently of everyone else’s.
3. Each bidder receives his individual valuation, $v_i$.

4. All bidders enter the auction and submit a bid, $b_i$. This bid may be a competitive or collusive bid, depending on what bidders decided in stage 2.

5. The seller decides who wins the auction. With probability $\lambda$, the seller chooses the winner randomly, and each bidder has a probability $\theta_i$ of winning the auction regardless of his bid; with probability $(1 - \lambda)$ the seller allocates the good to the bidder who submitted the highest bid.

We solve this model by backward induction: we first solve for the bidding strategy $(b_i)$ in stage 4 as a function of $\lambda$, $\theta_i$ and $v_i$. This bidding strategy is either competitive (Section 3) or collusive (Section 4). Given these two possible strategies, we then see which is more profitable before the valuations arrive (this is the decision of stage 2; Section 5.1). Finally, we solve for the seller’s optimal choice of $\lambda$ and $\theta_i$ ($\forall i$) which he announces in stage 1 (Section 5.3), knowing that all the decisions on the subsequent stages are conditional on this announcement.

3 The competitive auction

In this auction setup, where the seller is allowed to cheat on the auction rules, assuming all bidders bid competitively and do not form a cartel, the optimal bid must be lower than the optimal bid when the seller never cheats. Note that if the seller cheats with positive probability, each bidder will win the auction with some probability $\theta_i$, regardless of his bid. Hence, optimally, all bidders incorporate this in their optimal bid, and they all bid strictly less than in an auction where the seller never cheats.

To prevent confusion in our terminology, we define:

**Definition 1** A purely competitive auction is an auction where the seller never cheats ($\lambda = 0$).

**Definition 2** A competitive auction is an auction where the seller may cheat ($\lambda > 0$).

We now prove the following Proposition:

**Proposition 3** In a competitive auction (where the seller cheats with positive probability), and when no cartel is formed, each bidder $i$ will bid strictly less than in a purely competitive auction (in which the seller never cheats).
Proof. In order to prove this Proposition, we have to solve equation (3) assuming every bidder has received his valuation (so that \( v_i \) is known to every \( i \)). \( h_i(b_i) \), given by equation (4), is the probability of winning with a valuation \( v_i \) when the seller does not cheat, provided we assume the bidding function \( b_i(v_i) \) to be strictly increasing (we later on show that this is in fact true in the symmetric equilibrium). Thus, the probability of winning with a bid \( b_i \) is equal to the probability of the remaining \( (n-1) \) bidders having a valuation lower than \( v_i \), because \( b_i(.) \) is strictly increasing and symmetric. Let us also denote the inverse of \( b_i(v_i) \) by \( \psi(b_i) = b^{-1}(b_i) = v_i \). Hence, 

\[
h_i(b_i) = F(v_i)^{n-1} = F(\psi(b_i))^{n-1}.
\]

We can rearrange equation (3) making use of the notation we have just introduced:

\[
\pi_i = (v_i - b_i) \left( \lambda \theta_i + (1 - \lambda) F(\psi(b_i))^{n-1} \right)
\]  

(5)

The maximum of equation (5) must satisfy the first order condition, \( \partial \pi_i / \partial b_i = 0 \), which yields:

\[
-\left( \lambda \theta_i + (1 - \lambda) F(\psi(b_i))^{n-1} \right) + (v_i - b_i) (1 - \lambda) \left[ F(\psi(b_i))^{n-1} \right]' = 0
\]  

(6)

Notice that \( \left[ F(\psi(b_i))^{n-1} \right]' = (n - 1) F(\psi(b_i))^{n-2} f(\psi(b_i)) \psi'(b_i) \). and using \( \psi(b_i) = v_i \) and \( \psi'(b_i(v_i)) = 1/b'_i(v_i) \) we can rearrange it to yield:

\[
\left[ \theta_i \rho + F(v_i)^{n-1} \right] b'_i(v_i) + (n - 1) F(v_i)^{n-2} f(v_i) b_i(v_i) = v_i (n - 1) F(v_i)^{n-2} f(v_i)
\]  

(7)

which is a differential equation, where \( \rho = \lambda / (1 - \lambda) \). The two terms in the left hand side of this equation are the derivative of \( \left( \theta_i \rho + F(v_i)^{n-1} \right) b_i(v_i) \), so we can integrate both sides. This will yield:

\[
\left( \theta_i \rho + F(v_i)^{n-1} \right) b_i(v_i) - \left( \theta_i \rho + F(0)^{n-1} \right) 0 = \int_{0}^{v_i} v_i (n - 1) F(v_i)^{n-2} f(v_i) dv_i
\]  

(8)

The lower limit of the integral is 0 because all bidders are assumed to bid (even when \( v_i = 0 \)); such a bidder cannot receive a negative payoff, hence when his valuation is \( v_i = 0 \) he must bid \( b_i(0) = 0 \). The latter is the boundary condition and it is now straightforward to find the solution, which we denote by \( b^C(v_i) \) (we have dropped the subscript because the equilibrium is symmetric27):

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27The probability of winning the auction, \( h_i(b_i) \), was derived by assuming that \( b_i(v_i) \) was increasing and symmetric (i.e. the same function for all bidders); hence, this expression is the best response against itself, and it is a symmetric equilibrium.
\[ b^C(v_i) = \frac{1}{\theta_i \rho + F(v_i)^{n-1}} \left[ \int_0^{v_i} v_i (n - 1) F(v_i)^{n-2} f(v_i) dv_i \right] \quad (9) \]

This expression is similar to the one obtained by Milgrom and Weber (1982) in their general model, and we can integrate it by parts to yield:

\[ b^C(v_i) = \frac{1}{\theta_i \rho + F(v_i)^{n-1}} \left[ F(v_i)^{n-1} v_i - \int_0^{v_i} F(v_i)^{n-1} dv_i \right] \quad (10) \]

It is straightforward to check that \( b^C(v_i) \) is strictly increasing in \( v_i \), as assumed previously. The solution of a first-price auction with no cheating yields an optimal bid of (see, for example, Riley and Samuelson (1981) or Matthews (1995)):

\[ b^{PC}(v_i) = v_i - \frac{\int_0^{v_i} F(v_i)^{n-1} dv_i}{F(v_i)^{n-1}} \quad (11) \]

which is the result we obtain when we set \( \lambda = 0 \) (\( \rho = 0 \)) in equation (10) and where \( PC \) stands for Purely Competitive auction (with no cheating). To prove the Proposition, the expression in equation (11) must be less or equal than the one in (9) or (10), i.e. \( b^F(v_i) \leq b^{PC}(v_i) \), \( \forall v_i \). For this to be verified, the following expression must hold:

\[ \theta_i \rho \left( b^{PC}(v_i) \right) \geq 0 \quad (12) \]

which is true for any \( \theta_i \) or \( \rho \) and because \( b^{PC}(v_i) \geq 0 \).

It is rather intuitive to understand why this is so. If a bidder is faced with the threat of cheating by the seller, which means that there is a probability \( \lambda \) that the winning bid is not necessarily the highest, and \( \theta_i \) is the probability of bidder \( i \)'s bid being chosen, then the benefits of aggressive bidding are lower, as raising one’s bid only affects the probability of winning in the event that the seller does not cheat. Because with some probability \( \lambda \) it is likely that the highest bid is not the winning bid, all bidders will lower their bids from the purely competitive level. In a symmetric equilibrium, and with no cartel formation, the seller is worse off by cheating, because bidders will respond by lowering their optimal bids. And the higher is \( \theta_i \) or \( \lambda \), the lower will the optimal bids be, and hence the lower will the seller’s payoff be if he does cheat.
Corollary 4 When the seller cheats with probability \( \lambda = 1 \), the optimal bid by bidder \( i \) is 0, i.e. if \( \lambda = 1 \), \( b^C(v_i) = 0 \).

**Proof.** Substitute \( \lambda = 1 \) into equation (5). The expected profit of bidder \( i \) becomes:

\[
\pi_i = (v_i - b_i) \theta_i
\]  
(13)

\( \theta_i \) is exogenous to bidder \( i \) (it is set by the seller), and hence the bid which maximises equation (13) is the lowest possible bid, \( b^C(v_i) = 0 \).

Alternatively, substitute \( \lambda = 1 \) into equation (9) or equation (10). \( \lambda = 1 \) implies that \( \rho = \infty \), and hence \( b^C(v_i) = 0 \).  

When a bidder is faced with a seller who cheats with probability \( \lambda = 1 \), the winner of the auction will not be the highest bidder: instead, the winner will be chosen randomly because all submitted bids are equal. Hence, there is no gain in submitting a high (purely competitive) bid because the seller will choose the winner in an alternative way. Thus, bidder \( i \) prefers to bid 0 and let the seller choose the winner by whatever way he pleases. On the other hand, if \( \lambda = 0 \), the probability of the winner being the highest bidder approaches 1. This implies that, when \( \lambda = 0 \), the optimal bid in equation (9) is equal to the optimal bid in a purely competitive auction given by equation (11).

When the seller can cheat, the competitive profit of bidder \( i \) is (where \( b^C(v_i) \) is given by equation (10)):

\[
\pi_i^C(v_i) = v_i - b^C(v_i)
= \frac{1 - \lambda}{\lambda \theta_i + (1 - \lambda) F(v_i)^{n-1}} \left( \frac{\lambda}{1 - \lambda} \theta_i v_i + \int_0^{v_i} F(v_i)^{n-1} dv_i \right)
\]  
(14)

This bidder’s expected profit (given \( v_i \)) is given by the above equation, multiplied by the probability of winning the auction: \( \left[ \lambda \theta_i + (1 - \lambda) F(v_i)^{n-1} \right] \). Hence, the expected profit is given by:

\[
\pi_i^C(v_i) = \lambda \theta_i v_i + (1 - \lambda) \int_0^{v_i} F(v_i)^{n-1} dv_i
\]  
(15)

\( ^{28} \)The bid function, \( b(v_i) \), is increasing and continuous; this implies that the corner solution is equal to the interior solution evaluated at the point \( \lambda = 1 \).
Ex ante, before valuations are known, the competitive profits become\(^{29}\):

\[
E \left[ \pi^C_i (v_i) \right] = \int_0^\infty \pi^C_i (v_i) \cdot f (v_i) \, dv_i \\
= -\pi^C_i (v_i) \cdot (1 - F (v_i)) \bigg|_0^\infty + \int_0^\infty \frac{d\pi^C_i}{dv_i} (1 - F (v_i)) \, dv_i \\
= 2 \theta_i \int_0^\infty (1 - F(v_i)) \, dv_i + (1 - 2) \int_0^\infty (1 - F(v_i)) F(v_i)^{n-1} \, dv_i
\]

(16)

This simplification is made possible because \(-\pi^C_i (v_i) \cdot (1 - F(v_i)) \bigg|_0^\infty = 0\). By contrast, the ex ante expected profits of a purely competitive auction (when the seller never cheats) are given by:

\[
E \left[ \pi^{PC}_i (v_i) \right] = \int_0^\infty (1 - F(v_i)) F(v_i)^{n-1} \, dv_i
\]

(17)

Note how the expression in equation (16) is a linear combination of the expected purely competitive profits (with probability \((1 - \lambda)\) the winner is the highest bidder) and the expected profits when the seller cheats (which bidder \(i\) expects to receive with probability \(\lambda \theta_i\)).

4 The effects of cartel formation

Let us now analyse a cartel’s optimal strategy in the context of our model. A cartel, if formed, will maximise total ex ante expected profits (before bidders receive their valuations). We assume that all bidders become members of the cartel, if it is formed, and we put aside the problem of redistribution inside the cartel. This is quite naturally a very important matter. Many cartels break down because of unhappy members (“whistle-blowers”) after the redistribution of profits. But as explained in Section 2, we assume a weak cartel, which is not allowed to make transfers between its members, in the manner of McAfee and McMillan (1992).

Before the auction takes place, the cartel members meet and are asked to reveal their valuations (which will be their true valuations, once we assume incentive compatibility). Then the cartel mechanism suggests a bid to every member, which is in the member’s best interest to follow, given incentive compatibility. We also assume to be common knowledge that the seller awards the good randomly (with equal probabilities) when there is a tie between two or more bidders for the highest bid. This is the same environment as that of McAfee and McMillan (1992).

\(^{29}\)We assume \(F (\bar{v}) = 1\) to be the constant of integration, as McAfee and McMillan (1992) did in the integration by parts of (18).
McAfee and McMillan (1992) have shown that a weak cartel’s optimal strategy is to have all members submitting the lowest possible bid, 0. In particular, this outcome does not require explicit coordination between the members. The *ex ante* (before valuations are known) expected profit of a cartel member is given by (see Theorem 1, McAfee and McMillan (1992)):

\[
E \left[ \pi^C_i (v_i) \right] = \left[ \int_0^\infty v_i f (v_i) \, dv_i \right] \frac{1}{n} \tag{18}
\]

where \(1/n\) is the *ex ante* probability of bidder \(i\) winning the auction given that all cartel members submit the same bid (0). In other words, \(1/n\) is bidder \(i’s\) probability of winning the auction given that he is a cartel member.

We can integrate by parts the expression in equation (18), and this yields\(^3\):

\[
E \left[ \pi^C_i (v_i) \right] = \left[ \int_0^\infty (1 - F (v_i)) \, dv_i \right] \frac{1}{n} \tag{19}
\]

This is the *ex ante* expected profit of a cartel member. McAfee and McMillan (1992) have proved that for distributions such that

\[
H (v) = [1 - F (v)] / f (v) \tag{20}
\]

is decreasing (i.e. \(H' (v) < 0\)), this scheme (to have all bidders submitting the same bid, 0) is optimal, and hence it yields a higher expected payoff (for the bidders) than a purely competitive auction\(^3\) (when the seller never cheats). Most distributions have this property\(^3\). We will assume that this property applies to our model, and hence a cartel member can expect higher profits from being a member than if he was submitting a purely competitive bid.

---

\(^3\)In this integration by parts, we have assumed the constant of integration to be \(F (\infty) = 1\), in a way similar to McAfee and McMillan (1992). The expression in equation (18) can be rearranged as (using the constant of integration):

\[
E \left[ \pi^C_i (v) \right] = \left[ \int_0^\infty (-v) (-f (v)) \, dv \right] \frac{1}{n} = \left[ \int_0^\infty ((F (v) - F (v)) (-v)) \, dv \right] \frac{1}{n} = \left[ \int_0^\infty (1 - F (v)) \, dv \right] \frac{1}{n}
\]

And this is the expression in equation (19).

---

\(^3\)If \(H' (v) \geq 0\), McAfee and McMillan (1992) show that the cartel can never improve on the profits accrued from a purely competitive auction.

\(^3\)McAfee and McMillan (1992) mention the exponential distribution as being one of the exceptions.
5 Favouritism prevents cartel formation

Can the seller choose the values of $\lambda$ and $\theta_i$ in such a way as to disrupt a cartel, i.e. to persuade one of the cartel members to sign a contract with him and bid outside the cartel? In other words, will favouritism generate a higher payoff for the favourite bidder outside the cartel (and hence make him defect)? If the answer to this question is “yes”, then one must ask whether one defection is sufficient to disrupt the cartel. This Section will address these questions, and show that they are both true: the seller can manipulate $\lambda$ and $\theta_i$ in such a way as to make one bidder better off outside the cartel, and this defection is sufficient to disrupt it, i.e. one defection is sufficient to break down the cartel. Given these results, we then focus on the optimal choice of $\lambda$ and $\theta_i$ from the seller’s point of view.

5.1 Favouritism generates incentives for defection

Take a particular bidder $i$ (who is a cartel member). What would make this bidder defect from the cartel? His (ex ante) expected profit from being a cartel member is given by equation (19), whereas his (ex ante) profit from bidding competitively is given by equation (16). If the seller can select $\lambda$ and $\theta_i$ such that:

$$E \left[ \pi^C_i (v_i) \right] \geq E \left[ \pi_i^{CA} (v_i) \right]$$ (21)

then bidder $i$ would prefer to bid competitively and defect the cartel. Note that a defection by bidder $i$ implies the following:

- bidder $i$’s profits from defecting must be larger than bidder $i$’s profits in the cartel;
- in assessing the profits from defection, bidder $i$ must anticipate the response of other cartel members.

Our next Proposition shows that values for $\lambda$ and $\theta_i$ which satisfy the above inequality always exist, and hence the seller can always generate incentives for a given bidder $i$ to defect the cartel and bid competitively.

**Proposition 5** For a sufficiently large $\lambda$, there always exists a level of favouritism ($\theta_i$) which makes bidder $i$ defect the cartel and bid competitively.
Proof. Suppose the seller announces that bidder $i$ is his favourite bidder, and hence $\theta_i = 1$. This implies that if the seller cheats, he will pick bidder $i$ as the winner with probability $\lambda$. This also implies that $\theta_{-i} = 0$, for all bidders other than $i$, because $\sum_{i=1}^{n} \theta_i = 1$.

To prove the Proposition, at least one pair of values for $\lambda$ and $\theta_i$ must satisfy the inequality $E[\pi^C_i (v_i)] \geq E[\pi^{CA}_i (v_i)]$. Plugging $\lambda = 1$ and $\theta_i = 1$ into the inequality (and using the expressions obtained previously in equations (19) and (16)), we get:

$$E[\pi^C_i (v_i)] = \lambda \theta_i \int_0^\sigma (1 - F(v_i)) \, dv_i + (1 - \lambda) \int_0^\sigma (1 - F(v_i)) F(v_i)^{n-1} \, dv_i$$

$$= \int_0^\sigma (1 - F(v_i)) \, dv_i$$

$$\geq E[\pi^{CA}_i (v_i)]$$

$$= \frac{1}{n} \int_0^\sigma (1 - F(v_i)) \, dv_i$$

and hence there exists at least one pair of values for $\lambda$ and $\theta_i$ ($\lambda = 1$ and $\theta_i = 1$) such that bidder $i$ prefers to bid competitively\(^\text{33}\). Note that the assumption of $\lambda = 1$ is not necessary. As long as there are $n > 1$ bidders, the inequality holds for a $\lambda$ (or $\theta_i$) strictly lower than 1. 

This result is extremely important, because it proves that the seller can generate incentives for one cartel member to defect from the cartel and bid competitively instead. Also, note the importance of the contract signed between the seller and the favourite bidder. If the favourite bidder could accept the terms of the contract ($\lambda$ and $\theta_i = 1$) and still remain in the cartel, then the cartel’s optimal strategy would be unchanged. Each bidder would still submit a bid of 0, but bidder $i$ would face an increased probability of winning (with probability $\lambda$ the seller would award the good to him). Because the cartel is assumed not to be able to affect $\lambda$ or $\theta_i$ (which are assumed to be exogenous), McAfee and McMillan’s (1992) optimal strategy would be unchanged. However, if the contract signed with bidder $i$ (the favourite bidder) entails as a condition that bidder $i$ must bid outside the cartel, then Proposition 5 shows that the terms of the contract can be incentive compatible, i.e. the contract binds bidder $i$ to bidding outside the cartel if he accepts it and the terms of the contract are designed in such a way that he does accept it in equilibrium.

More importantly, this result introduces a trade-off between $\lambda$ and $\theta_i$. In an auction with $n > 1$ bidders, the seller can generate incentives for bidder $i$ to defect in one of two ways: he can increase $\theta_i$ or $\lambda$, i.e. he can increase the probability of favouring a particular bidder (when he cheats) or

\(^{33}\)The inequality holds strictly for an auction with $n > 1$ bidders.
he can increase the overall probability of cheating. When $\theta_i$ and $\lambda$ are close to 1, the inequality in equation (22) is always satisfied: bidder $i$ will always prefer to bid competitively and enter the contract with the seller.

Moreover, the inequality in equation (22) assumes that all of bidder $i$'s opponents bid competitively as a best response to bidder $i$'s defection (in Section 5.2 we prove that this is in fact the case). Hence, for values of $\theta_i$ and $\lambda$ which satisfy equation (22), any other response by the cartel members to bidder $i$'s defection will increase bidder $i$'s profits. For example, if the cartel does not change its strategy, and recommends every member (i.e. all bidders except bidder $i$) to bid 0, then bidder $i$ will always win the auction and receive a payoff higher than $E[\pi^C_i(v_i)]$ in equation (22). Therefore, the conditions under which equation (22) is satisfied are the most conservative possible. In Section 5.2, we show that the best response by all cartel members is to bid competitively, as a response to bidder $i$'s defection, and hence abandon the cartel altogether. Therefore, a defection by the favourite bidder is guaranteed when equation (22) is satisfied, i.e. in the equilibrium obtained when other bidder's best responses are taken into account.

When $\theta_i = 1$ (when the seller favours bidder $i$ with probability 1 when he cheats), there exists a minimum value of $\lambda$ (which we define as $\lambda_i$) such that the inequality of equation (22) is still satisfied. Similarly, when $\lambda = 1$, there exists a minimum value of $\theta_i$ (which we define as $\theta_i$) such that the inequality is also satisfied. This is an early hint that an optimal choice of $\lambda$ and $\theta_i$ must exist, i.e. the seller can choose a pair $(\lambda, \theta_i)$ which maximises his revenue.

Knowing that the seller can manipulate $\lambda$ and $\theta_i$ in such a way as to make bidder $i$ prefer to bid competitively is important. But is this defection enough to disrupt the cartel?

### 5.2 Favouritism disrupts a cartel

By selecting a pair $(\lambda, \theta_i)$ which satisfies equation (22), the seller generates incentives for a particular bidder to defect the cartel, and bid competitively. Most importantly, any $\lambda$ which satisfies equation (22) is strictly lower than 1;\(^{34}\) this implies that if bidder $i$ defects and bids competitively, his competitive bid is higher than 0: $b^C(v_i) > 0$ (see Corollary 4). The cartel’s strategy of having all members bidding 0 is no longer optimal: if all bidders other than $i$ bid 0, and bidder $i$ bids competitively, bidder $i$ will always win the auction when the seller does not cheat (which happens with probability $(1 - \lambda)$), because $b^C(v_i) > 0$. What, then, is the cartel’s optimal strategy when one bidder defects the cartel and bids competitively?

\(^{34}\)As long as there are at least 2 bidders in the auction - see Proposition 5.
Our next result shows the cartel’s optimal strategy, when bidder \( i \) defects and bids competitively, is to have all its members bidding competitively as well. The cartel cannot improve on the competitive profits. Hence, the cartel’s function as a coordination mechanism is no longer necessary: bidders can bid competitively without its existence. This proves that favouritism disrupts the cartel: all its members will prefer to bid competitively if one of its members defects.

**Proposition 6** If the pair \((\lambda, \theta_i)\) induces bidder \( i \) to defect the cartel and bid competitively, then all other bidders \(-i\) will also prefer to defect and bid competitively. Hence, the cartel is destabilised.

**Proof.** Proposition 5 proves that as long as \( n > 1 \), any pair \((\lambda, \theta_i)\) which induces bidder \( i \) to defect implies \( \lambda < 1 \). Hence, when bidder \( i \) defects, he will bid \( b^C(v_i) > 0 \). Bidding 0 is no longer optimal for all other bidders \((-i)\), because if they bid 0 they would never win the auction when the seller does not cheat (when the winning bid is the highest bid)\(^{35}\).

Given that bidder \( i \) is bidding competitively, all other bidder’s best strategy is also to bid competitively. This is an implication of the symmetric equilibrium argument in Milgrom and Weber (1982), also used in Proposition 3. The best response to \( b^C(v_i) \) is \( b^C(v_{-i}) \), given in equation (10). A bid \( b < b^C(v_{-i}) \) would yield a higher payoff but a lower probability of winning the auction; a bid \( b > b^C(v_{-i}) \) would increase the probability of winning but yield a negative payoff. The bid \( b^C(v_{-i}) \) maximises the expected payoff given that bidder \( i \) is bidding \( b^C(v_i) \), and hence is the best response to this strategy.

In order to implement \( b^C = (b^C(v_i), b^C(v_{-i})) \) (with \( b^C(v_{-i}) \) being the vector of equilibrium bids for all bidders except \( i \)), a cartel (or coordination mechanism) is no longer necessary. Bidders can implement the optimal strategy without coordination (and hence without the presence of a cartel). \( \blacksquare \)

This implies that one defection from the cartel is sufficient for all its members to defect as well. By favouring one particular bidder when he cheats, the seller is implicitly generating incentives for all cartel members to defect and bid competitively. But as we have seen already, there exists a range of values for \((\lambda, \theta_i)\) which disrupt a cartel. Which is the optimal choice for the seller?

### 5.3 The seller’s optimal choice of \( \lambda \) and \( \theta_i \)

Section 5.1 has shown that the seller can select a pair \((\lambda, \theta_i)\) which will make bidder \( i \) prefer to bid competitively and defect the cartel; Section 5.2 has proved that this defection is sufficient to

\(^{35}\)If \( \lambda = 1 \), all bidders will bid \( b^C(v_i) = 0 \). In this case, the competitive outcome is the same as with a cartel. Hence, even if the cartel is disrupted, all bidders behave in the same way as if the cartel was still present.
disrupt the cartel. But which pair \((\lambda, \theta_i)\) generates the maximum revenue for the seller?

In Section 5.1 we have noted that there exists a range of values for \(\lambda\) and \(\theta_i\) which will disrupt the cartel. In particular, when \(\theta_i = 1\) there exists a minimum value of \(\lambda = \overline{\lambda}\) which generates cartel disruption; similarly, when \(\lambda = 1\), there exists a minimum value of \(\theta_i = \underline{\theta}_i\) which also generates cartel disruption.

The seller wants to maximise the expected price in the auction, which is a function of \(\lambda\) and \(\theta_i\):

\[
P(\lambda, \theta_i) = \sum_{i=1}^{n} \lambda \theta_i (b^C(v_i)) + (1 - \lambda) \left[ \max \left( b^C(v_i); b^C(v_{-i}) \right) \right]
\]  

Our next result shows that the optimal choice for the seller is \((\lambda, \theta_i) = (\overline{\lambda}, 1)\), i.e. the expected price is maximised when the seller favours one particular bidder with probability 1 (when he cheats).

**Proposition 7** The pair \((\lambda, \theta_i)\) which maximises the expected price in the auction is \((\lambda, \theta_i) = (\overline{\lambda}, 1)\).

**Proof.** Let bidder \(i\) be the favourite bidder. His competitive bid when \(\lambda = 1\) (and \(\theta_i = \theta_j\)) is \(b^C(v_i) = 0\) (see Corollary 4); all his opponents will also bid \(b^C(v_{-i}) = 0\). The expected price for the seller is:

\[
P(\lambda = 1, \theta_i = \theta_j) = \sum_{i=1}^{n} \lambda \theta_i (b^C(v_i)) + (1 - \lambda) \left[ \max \left( b^C(v_i); b^C(v_{-i}) \right) \right] = 0
\]  

However, when the seller announces instead \(\lambda = \overline{\lambda}\) and \(\theta_i = 1\), bidder \(i\) will bid (where \(\underline{\theta} = \overline{\lambda}/(1 - \overline{\lambda})\)):

\[
b^C(v_i) = \frac{1}{\underline{\theta} + F(v_i)^{n-1}} \left[ F(v_i)^{n-1} v_i - \int_{0}^{v_i} F(v_i)^{n-1} dv_i \right] \geq 0
\]  

whereas all bidders \(-i\) will bid as in a purely competitive auction (note that when \(\theta_i = 1\), \(\theta_{-i} = 0\), and hence \(b^C(v_{-i}) = b^{PC}(v_{-i})\)):

\[
b^C(v_{-i}) = v_{-i} - \frac{1}{F(v_{-i})^{n-1}} \int_{0}^{v_{-i}} F(v_{-i})^{n-1} dv_{-i} \geq 0
\]
In this case, the expected price for the seller when $\lambda = \lambda$ and $\theta_i = 1$ is given by:

$$P(\lambda = \lambda, \theta_i = 1) = \sum_{i=1}^{n} \Delta \theta_i (b^C(v_i)) + (1 - \lambda) \left[ \max (b^C(v_i); b^C(v_{-i})) \right]$$  \hspace{1cm} (27)

To prove this Proposition, we need to show that the expression in equation (27) is larger than the expression in equation (24). Let us start by analysing the first term in both equations. From Proposition 5 we know that $1/n \leq \lambda \leq 1$ and $1/n \leq \theta \leq 1$ for the favourite bidder to accept to defect. Let

$$K = F(v_i)^{n-1} v_i - \int_0^{v_i} F(v_i)^{n-1} dv_i$$

$$\geq 0 \hspace{1cm} (28)$$

and

$$L = \left[ \theta_i \lambda + (1 - \lambda) F(v_i)^{n-1} \right]^2$$

$$\geq 0 \hspace{1cm} (29)$$

Differentiating $b^C(v_i)$ with respect to $\lambda$ yields:

$$\frac{\partial b^C(v_i)}{\partial \lambda} = \frac{K}{L} (-\theta_i)$$  \hspace{1cm} (30)

and differentiating $b^C(v_i)$ with respect to $\theta_i$ yields:

$$\frac{\partial b^C(v_i)}{\partial \theta_i} = \frac{K}{L} (\lambda^2 - \lambda)$$ \hspace{1cm} (31)

Thus, within the ranges of $\lambda$ and $\theta_i$ which generate cartel disruption ($1/n \leq \lambda \leq 1$ and $1/n \leq \theta \leq 1$) we obtain:

$$\frac{K}{L} (-1) \leq \frac{\partial b^C(v_i)}{\partial \lambda} \leq \frac{K}{L} \left( -\frac{1}{n} \right)$$ \hspace{1cm} (32)

$$\frac{K}{L} \left( \frac{1}{n^2} - \frac{1}{n} \right) \leq \frac{\partial b^C(v_i)}{\partial \theta_i} \leq 0$$ \hspace{1cm} (33)
And therefore within the range of values which generate cartel disruption:

$$\frac{\partial b^C(v_i)}{\partial \lambda} < \frac{\partial b^C(v_1)}{\partial \theta_i} \leq 0$$  \hspace{1cm} (34)

Hence, choosing to manipulate \( \lambda \) (i.e. increase it above \( \Lambda \)) is more costly in terms of lost revenue than choosing to manipulate \( \theta_i \) (i.e. increase it above \( \theta_i \)). Reversing the argument, we can see that reducing \( \lambda \) from 1 (in equation (24)) and increasing \( \theta_i \) from 1 will lead to an increase in \( b^C(v_i) \). This increase is limited by the minimum value of \( \lambda (\Lambda) \) which still generates cartel disruption. Thus, the seller strictly prefers to announce \( \theta_i = 1 \). This may be seen as the lowest possible cost for the seller (reduction in revenue as compared to the purely competitive auction) of cheating.

To finish the proof, we need to show that the second term of equation (27) is also larger than the second term in equation (24). All the elements of \( b^C(v_{-i}) \) must increase as \( \theta_i \) approaches 1 (and \( \lambda \) approaches \( \Lambda \)). From equations (30) and (31), we know that the derivatives of \( b^C(v_{-i}) \) with respect to \( \lambda \) and \( \theta_{-i} \) are negative \( \frac{\partial b^C(v_{-i})}{\partial \lambda} < 0 \) and \( \frac{\partial b^C(v_{-i})}{\partial \theta_{-i}} < 0 \); note that as \( \theta_i \) approaches 1, \( \theta_{-i} \) decreases and approaches 0. Hence, \( b^C(v_{-i}) \) increases as both \( \lambda \) and \( \theta_{-i} \) decrease. In the limit, when \( \theta_i = 1 \) (and hence \( \theta_i = 1 \)), \( b^C(v_{-i}) \) is the purely competitive bid (regardless of \( \lambda \)). This proves that every element of \( b^C(v_{-i}) \) increases with \( \theta_i \).

Additionally, \( b^C(.) > b^C_i(.) \geq 0 \) for any value of \( \lambda \) and \( \theta_i \) which generates cartel disruption. As \( \theta_i \) increases (and approaches 1), the probability of the expected price being the maximum of \( b^C(v_{-i}) \) increases: all the elements of \( b^C(v_{-i}) \) increase relative to \( b^C(v_i) \), thus increasing the probability that they contain the maximum\(^{36}\). This proves that \( P(\lambda, \theta_i) \) is increasing with \( \theta_i \) and hence announcing \( \theta_i = 1 \) and \( \lambda = \Lambda \) is the optimal strategy for the seller. \( \blacksquare \)

This optimal strategy provides some very interesting insights. The seller benefits from cheating with the lowest probability which still disrupts the cartel \( (\lambda = \Lambda) \) but favouring one particular bidder with probability 1 when he does cheat \( (\theta_i = 1) \). In our model, increasing the probability of cheating is more costly than increasing the probability of favouring a particular bidder when he does cheat: increasing the probability of cheating induces all bidders to lower their bids. By contrast, increasing the probability of favouring a particular bidder will induce this bidder to lower his bid, but will also induce all other bidders to increase theirs. The total price reduction of increasing the overall probability of cheating is higher than the reduction from favouring one particular bidder.\(^{36}\)

\(^{36}\)As \( \lambda \) decreases, all the elements of \( b^C(v_{-i}) \) and \( b^C(v_i) \) increase, but the former increase relatively more than the latter. This induces a shift in the probability distribution which attaches more weight to elements of \( b^C(v_{-i}) \), thus increasing the probability that \( b^C(v_{-i}) \) contains the maximum.
Thus, the optimal strategy is to favour one particular bidder with the highest possible probability, and to set the lowest possible overall probability of cheating which still disrupts a cartel (and which is strictly positive).

In our model, cheating is a means to an end: in the presence of a cartel, the seller chooses to cheat and favour a particular bidder in the auction as a way to disrupt the cartel. However, the expected price when he does cheat is always lower than the expected price in a purely competitive auction (where all bidders bid competitively and the seller never cheats). Cheating is a second-best solution, but the one which maximises the seller’s expected payoff when there is a threat of cartel formation.

6 Conclusion

McAfee and McMillan (1992) have shown how serious cartels should be taken: a (weak) cartel’s optimal strategy is to have all members submitting the reserve price, and this reduces the seller’s expected revenue. Our model allows the seller to respond to such a threat, by cheating on the auction rules and allocating the item for sale to a pre-specified bidder, regardless of whether his bid was actually the highest. We show that when no cartel is present, such a strategy reduces the seller’s expected revenue compared to a purely competitive auction, in which the highest bidder always wins the auction. However, if a cartel is active, this strategy is effective in generating incentives for the pre-specified bidder (who we call the seller’s favourite) to defect from the cartel, and we prove that this single defection is sufficient to break it down.

Several combinations of the seller’s overall probability of cheating and a particular bidder’s probability of being allocated the item (when the seller does cheat) are effective in disrupting the cartel. However, we prove that the optimal strategy is to allocate the item with probability one to a pre-specified bidder when the seller decides to cheat (favouritism). Not only does this strategy succeed in generating incentives for this bidder to defect the cartel, but it also leads to increased bids from all other (former) cartel members. In this equilibrium, only the favourite (pre-specified) bidder will bid less than he would have in a purely competitive auction, and all other bidders bid exactly as in a purely competitive auction; we prove that this seller’s strategy is optimal, i.e. it generates the highest possible revenue (given that the seller cheats with positive probability when facing the threat of cartel formation), although it is lower than the revenue in a purely competitive auction (in which no cartel is formed and the seller cannot cheat).

Our model shows how cheating may be an effective tool against cartel formation, but, as we
have noted, cheating is usually associated with lobbying and corruption. The seller may attempt to favour a particular bidder for political reasons (lobbying) or simply because that bidder bribes the seller. Such actions usually contribute to the reinforcement of the lobbyist’s (or briber’s) market position, and thus stifle competition. Interestingly, our model shows how cheating can also be beneficial in collusive markets, by generating incentives for cartel disruption.

It is difficult to draw a line between these two contradictory objectives. In fact, it may actually be impossible to know exactly what the seller had in mind when he decided to cheat on the auction rules. But our model sheds light over the beneficial aspects of cheating, and suggests that it is not always a “bad” thing. In particular, as McAfee and McMillan (1992) have suggested, this is only one of the many possible actions a seller may take when faced with a cartel in an auction. And although is may not be the most popular (because of its common association to corruption and bribery), it is certainly effective.

In the future, it would be interesting to compare and rank (in terms of revenue raised) the several tools the seller can use to prevent cartel formation. It is our belief that cheating would rank high compared to other tools\textsuperscript{37}, especially in auctions with many bidders. Therefore, we feel our model has suggested a particularly effective means of disrupting a cartel. However, extending it to a multiperiod setting\textsuperscript{38} (in which a government auctions off a sequence of contracts over time) would perhaps provide some more interesting (and maybe more efficient) cartel fighting tools. In our model, the seller is able to prevent cartel formation at a cost (the lower revenue it receives because it threatens to cheat). But in a multiperiod model, it is possible that the seller does not want to incur in this cost in all auction periods. Alternative strategies may appear, in which the seller does not always prevent cartel formation (i.e. a cartel may operate during some periods), but which over time produce an expected revenue at least as high as when he cheats in every auction period.

References


\textsuperscript{37}See McAfee and McMillan (1992).

\textsuperscript{38}As in Feinstein, Block and Nold’s (1985) model.


