Discriminatory Limit Pricing

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Abstract

We consider a two-period framework where a multimarket incumbent firm faces, in one of the markets, a single potential entrant offering a differentiated product. The incumbent has private information about his production cost and may use both pre-entry prices as predatory signals. We find multiple pure strategy perfect bayesian equilibria. Using equilibrium refinements, we show that there is always a unique reasonable perfect bayesian equilibrium. Our results show that in some cases this unique equilibrium entails a downward distortion in both low cost incumbent’s pre-entry prices. Moreover, we show that this distortion is identical in both markets and increasing with the discount factor, the degree of product substitutability and the efficiency of the entrant.

Keywords: Entry Deterrence, Product Differentiation, Asymmetric Information, Discriminatory Pricing.


1 Introduction

Predation involves a dominant firm that sets low prices and thus sacrifices short-run profit in order to drive a rival out of the market or deter entry, thus achieving higher profit in the long-run – see Joskow and Klevoric (1979), Niels (1993), Tirole (1988) and Motta (2004). Thus one can

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either have models of predation where the incumbent firm forces exit or models where entry is discouraged.

Following McGee (1958) assessment of the rationality of predation, several models have been developed providing a convincing context where predation is a rational strategy. Most of these predation models are based on incomplete information. This is the case of signaling models, reputation models and long purse predation models. Predation signaling models’ type of reasoning was first developed by Milgrom and Roberts (1982). In this model, a potential entrant has imperfect knowledge about the incumbent’s production cost and the incumbent exploits this uncertainty in order to make the entrant believe that entry is unprofitable (limit pricing model). Reputation models explore the incentive to prey if the incumbent sells in different markets (see Kreps and Wilson (1982)) while long purse predation models consider frameworks where there are differences between firms’ financial constraints and thus the incumbent can survive longer using its deep pockets (see Benoit (1984)).

This paper focus on a monopolistic limit pricing model but uses a multimarket setup in order to analyze discriminatory limit pricing. Considering different independent geographical markets, the use of third-degree price discrimination is optimal and studying how entry is affected when the incumbent sets discriminatory prices to deter entry is a relevant and interesting issue.

We develop a two-period framework where a multimarket incumbent faces a single potential entrant in one of the two markets where he operates. The incumbent has private information about his production cost, which may either be low or high. In the first period, the incumbent sets his optimal third-degree discriminatory prices. The entrant observes the prices in both markets and decides whether to enter or not after updating his beliefs on the likelihood of the incumbent’s production cost being low. If entry occurs, firms compete in prices. Naturally, the entrant will use the price information of every market where the incumbent operates in order to infer incumbent’s production cost. Therefore, the incumbent may use pre-entry prices as

\footnote{Other contexts in which predation is feasible and rational are, for example, models with learning curves (Cabral and Riordan (1997)) and models with increasing returns (Ordover and Saloner (1989)).}

\footnote{Harrington (1987) and Bagwell and Ramey (1991) explore multiple incumbents’ setups. For simultaneous oligopolistic signaling see also Mailath (1989).}

\footnote{Under a spatial market model, Aguirre et al. (1998) explore the strategic choice of pricing policies to deter entry when there is asymmetric information about incumbent firm’s transportation costs. They show that the incumbent firm may use spatial price discrimination to deter entry.}
predatory signals for his production cost. Our aim is to study the impact of an entry deterrence strategy in a third-degree price discrimination signaling model. We look for the determinants of these discriminatory limit prices when firms offer differentiated products. Notice that, the use of differentiated products has not been widely explored in these setups. Considering linear demands, our framework enables the analysis of different relationships between monopoly markets’ demand parameters.

We analyze the pure strategy perfect bayesian equilibria and find multiple equilibria. This multiplicity is due to the lack of restrictions upon off-the-equilibrium path posterior beliefs. If, under the prior beliefs, the entrant’s expected profit is higher than his entry costs then there are no pooling equilibria but there are multiple separating perfect bayesian equilibria. Using the equilibrium refinement that the entrant should put zero probability on any type of incumbent using a dominated strategy, we show that there is a unique perfect bayesian equilibrium which survives this criterion: the least cost separating equilibrium. When, under the prior beliefs, the entrant’s expected profit is lower than his entry costs and the two types of incumbent’s production cost are not too different, there are multiple pooling and separating equilibria. However, if the low cost incumbent’s monopoly prices can not be supported as one of the pooling equilibria, the only perfect bayesian equilibrium that survives the domination criterion is also the least cost separating equilibrium. On the other hand, when the low cost incumbent’s monopoly prices can be supported as one of the pooling equilibria, there are multiple perfect bayesian equilibria that survive the intuitive criterion: the least cost separating equilibrium and a set of pooling equilibria in the line-segment from the low cost equilibrium prices in the least cost separating equilibrium to the low cost incumbent’s monopoly prices. Using Grossman and Perry (1986) criterion, we show that only one perfect bayesian equilibrium survives: the pooling equilibrium where both types of incumbents set the low cost incumbent’s monopoly prices. Therefore, there is always a unique perfect bayesian equilibrium which survives this equilibrium refinement upon off-the-equilibrium path posterior beliefs.

Our results show that, in the least cost separating equilibrium, the low cost incumbent’s efforts to deter entry may or may not lead to a downward distortion in pre-entry discriminatory prices with respect to the incumbent’s prices under a complete information setting. The low cost incumbent uses both pre-entry prices to signal low production cost. When there is a downward distortion in prices, the decrease from the low cost incumbent’s monopoly prices is the same in
both markets. We also explore the determinants of the downward distortion in pre-entry prices.

Other authors have explored the use of multiple signals to deter entry.\footnote{Bagwell and Ramey (1988) extend Milgrom and Roberts (1982) incomplete cost information model by allowing firms to use price and advertising as potential signals and show that, in a sequential equilibrium, low prices and high advertising are used. This paper is the closest to our analysis. Bagwell and Ramey (1990) explore a setup where the incumbent has private information concerning market’s demand and study sequential equilibria for different alternatives of what the incumbent prefers that the market be believed to have (high or low demand). When the incumbent prefers to be believed to operate in a low demand market, low prices and low advertising are used to signal low demand whereas when the incumbent prefers to be believed to operate in a high demand market, both high prices and advertising are used. In both cases, dissipative advertising is never used. This paper differs from this research as these authors consider that firms operate in only one market and sell homogeneous products. We show that when the incumbent firm sells in more than one market, he uses every pre-entry price to signal his production cost and deter entry. We also show that the degree of product substitutability has impact on the predatory signals.}

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Our setup can be applied to anti-predation regulation theory since it reveals differences in discriminatory pricing behavior when there are entry deterrence objectives from when there is pure third-degree price discrimination (within a complete information setting). This is an important issue since mistaking pure third-degree price discrimination as predation may inhibit the use of price discrimination. Our results show that there is a set of parameters’ values where there exists a downward distortion in pre-entry discriminatory prices and thus detection of predation would be possible. We show that the biggest distortion in prices occurs when products are highly substitutable.

Predatory dumping is also closely related to our analysis. If we consider the market where there is no entry as the domestic market and the other market as the external and potentially competitive market, the incumbent may set a lower price in the external market so as to deter entry (traditional definition of dumping). As we use a more general third-degree price discrimi-
nation model, this result is only valid for the set of parameters where demand in the domestic market is larger than demand in the external market. We extend the analysis to capture this scenario.

This paper is organized in four sections. In Section 2 we set up the model and our main assumptions. Perfect bayesian equilibria are analyzed in Section 3 where the separating equilibria and the pooling equilibria are studied. Section 4 extends the initial model. Section 5 concludes and suggests other possible extensions.

2 The Model

Consider a two-period model where an incumbent firm, $I$, operates in two distinct and independent geographical markets, $A$ and $B$. The incumbent can use discriminatory pricing in these markets. In the first period only this firm is operating. The incumbent faces a potential entrant firm, $E$, in market $A$ only. The incumbent’s constant marginal production cost can be high or low, denoted by $c \in \{\bar{c}, \underline{c}\}$, where $\bar{c} > \underline{c}$ and also $\underline{c} = 0$. To simplify notation we will drop the superscript $I$ for the remaining of this paper. However, one should keep in mind that when a price, quantity, profit function or marginal production cost has no superscript they represent incumbent’s variables. The incumbent’s production cost is private information. The entrant holds a prior belief, assumed to be common knowledge, concerning the probability, $\rho_o$, that the incumbent has low production cost, with $\rho_o \in [0, 1]$. Knowing his production cost, the incumbent sets first period prices in both markets. After observing the incumbent’s pre-entry price in each market, $p_A$ and $p_B$, the entrant updates his beliefs concerning the probability that the incumbent has low production cost. We denote the posterior belief that the incumbent has low production cost by $\rho(p_A, p_B)$, with $\rho \in [0, 1]$. Based upon this posterior belief, the entrant decides whether to enter market $A$. Since the entry decision is taken after the incumbent has set his first period price in each market, these pre-entry prices can be used to signal information about the type of incumbent’s production cost. Second period profits are discounted by $\delta$, with $\delta \in [0, 1]$.

If entry occurs in the second period, firms are price-setting duopolists in a differentiated product market. The entrant also has constant marginal production cost, $c^E$, where $c^E \geq 0$.}

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5We assume there is no arbitrage.
Let \( f^E \) denote the entrant’s cost of entering market \( A \), where \( f^E \geq 0 \). Both \( c^E \) and \( f^E \) are known by the incumbent. Both firms have null fixed costs.

We assume linear demand in both markets. This type of demand function can be derived from the consumer’s utility maximization problem using a quadratic utility function. To derive demand in market \( i \) when only the incumbent operates, we assume that the representative consumer’s preferences are given by:

\[
U(q_i) = q_i^o + a_i q_i - \frac{1}{2} b_i (q_i)^2 ,
\]

where \( q_i \) represents the quantity sold in market \( i \) by the incumbent, \( q_i^o \) represents all other market \( i \) products (with a price normalized to unity) and \( a_i, b_i \) are positive constants. The consumer’s budget constraint is given by \( M_i = p_i q_i + q_i^o \).

If there is entry in market \( A \), in the second period firms choose their prices simultaneously and independently. These firms offer differentiated products, i.e., firms’ products are imperfect substitutes. In market \( A \), the representative consumer’s utility function takes the form of a standard quadratic utility function (similar to Dixit (1979)):

\[
U(q_A, q^E) = q_A^o + a_A q_A + a_A q^E - \frac{1}{2} \left[ b_A (q_A)^2 + 2d_A q_A q^E + b_A (q^E)^2 \right] ,
\]

where \( q_A \) and \( q^E \) represent the quantity sold in market \( A \) by the incumbent and the entrant, respectively. All other market \( A \) products are represented by \( q_A^o \) (with a price normalized to unity) and \( a_A, b_A \) are positive constants. The degree of product substitutability can be measured by \( d_A \in [0, b_A] \) where products become completely differentiated as \( d_A \to 0 \). When \( d_A \to b_A \), products become closer substitutes implying intense price competition. We require that \( d_A < b_A \) in order to assure that each product price is more sensitive to a change in its product quantity than to that of the other firm’s product quantity. Let us define \( \tau \) as the ratio between \( d_A \) and \( b_A \), i.e., \( \tau = \frac{d_A}{b_A} \). Therefore the degree of product substitutability can also be measured by \( \tau \) with \( \tau \in [0, 1] \). Values of \( \tau \) close to 1 correspond to highly substitutable products whereas values of \( \tau \) close to 0 correspond to highly differentiated products. Intermediate values of \( \tau \) stand for an intermediate extent of product substitutability. In the remaining analysis we will use \( \tau \) to measure the degree of product substitutability. Notice that the utility function (2) assumes

\[\text{We assume that firm E’s entry cost in market B is so high that entry in this market would always be unprofitable.}\]
that the two products have symmetric effects on the consumer’s utility. The consumer’s budget constraint is given by \( M_A = p_Aq_A + p^E q^E + q^O \).

From constrained optimization of the utility functions given by (1), we obtain the inverse demand function in each market. In order to derive the Nash equilibrium of the price-game, we compute the demand function in each market. Thus, in market \( B \) we obtain:

\[
q_B = \frac{a_B}{b_B} - \frac{1}{b_B}p_B
\]

and in market \( A \) when there is a monopoly:

\[
q_A = \frac{a_A}{b_A} - \frac{1}{b_A}p_A.
\]

If the incumbent with production cost \( c \) sets a price \( p_i \), his monopoly profit in market \( i \) is given by:

\[
\pi_i(p_i) = (p_i - c) \left( \frac{a_i}{b_i} - \frac{1}{b_i}p_i \right) = - \frac{1}{b_i}p_i^2 + p_i \left( \frac{a_i + c}{b_i} \right) - \frac{a_i}{b_i}c.
\]

The unique maximizer of this profit function is the monopoly price in market \( i \). Let \( \pi_{im} \) and \( p_{im} \) be the monopoly prices in market \( i \) when the incumbent has high and low production costs, respectively. Let \( \pi_{im} \) and \( \pi_{im} \) be the monopoly profits (see computation details in Appendix A.1).

From constrained optimization of the utility function (2), demand functions under entry in market \( A \) are given by:

\[
q_A = \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2}p_A + \frac{d_A}{b_A^2 - d_A^2}p^E
\]

\[
q^E = \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2} - \frac{b_A}{b_A^2 - d_A^2}p^E + \frac{d_A}{b_A^2 - d_A^2}p_A.
\]

To simplify computations when there is a duopoly in market \( A \) we define:

\[
\alpha = \frac{a_A(b_A - d_A)}{b_A^2 - d_A^2}, \quad \beta = \frac{b_A}{b_A^2 - d_A^2} \quad \text{and} \quad \gamma = \frac{d_A}{b_A^2 - d_A^2}.
\]

and thus:

\[
q_A = \alpha - \beta p_A + \gamma p^E
\]

\[
q^E = \alpha - \beta p^E + \gamma p_A.
\]

Notice that when entry does not occur, this market has no product variety since \( q^E = 0 \). In this case, these consumer’s specifications are identical to when there is a monopoly in market \( A \).
With this demand specification, the quantity demanded of a given product depends negatively on its own price and positively on the price of the other product. As $\tau = \frac{d_A}{b_A} \to 1$, the price of the other product affects more and more the quantity demanded of a firm’s product. When we derive the Nash equilibrium of the price competition game where both firms have common knowledge of their costs, let $\bar{\pi}_{Ad}, \bar{\pi}^E$ and $\bar{\pi}_{Ad}, \bar{\pi}^E$ be the duopoly equilibrium profits of the incumbent and duopoly gross equilibrium profits of the entrant in market $A$ when the incumbent has high and low production costs, respectively (see computation details in Appendix A.2).

Finally, we assume the following relationships between market $B$ and market $A$’s parameters:

$$a_B = ka_A \quad (3)$$

$$b_B = b_A. \quad (4)$$

Parameter $k$ measures the degree of markets’ demand difference. Under no entry and assuming (3) and (4), when $k \to 1$ market $A$ and market $B$ are identical. When $k < 1$, demand in market $A$ is larger than demand in market $B$ and demand in market $A$ is less elastic than demand in market $B$ for a given market price. Conversely, when $k > 1$, demand in market $B$ is larger than demand in market $A$.

Our analysis focus on $k < 1$ only, thus demand in market $A$ is less elastic than demand in market $B$ for a given market price. For these $k$ values, let $M$ be the set of parameters’ values where all monopoly quantities (market $A$ and $B$) are non-negative and $D$ be the set of parameters’ values where all duopoly quantities are non-negative (see details in Appendix B). Notice that, within the latter set of parameters’ values, $\bar{\pi}_{Ad}, \bar{\pi}^E, \bar{\pi}_{Ad}$ and $\bar{\pi}^E$ are all non-negative. Therefore, we consider $k < 1$ and parameters’ values $\{\tau, c^E, k\} \in M \cap D$.

### 3 Perfect bayesian equilibria

In this section we analyze the pure strategy perfect bayesian equilibria of our signalling game. A perfect bayesian equilibrium (PBE) is fully characterized if we set strategies for each firm as a function of the information available at each decision point as well as beliefs for the entrant about the incumbent’s production cost. The beliefs must be consistent with the information
structure (using Bayes’ rule) and with the hypothesis that the given strategies were being set. Given beliefs, all strategies must be best response strategies.

In the next subsection we analyze separating equilibria and subsection 3.2 studies the pooling equilibria. The equilibria results are summarized in subsection 3.3 where, using equilibrium refinements, we are able to show that there always exists a unique reasonable PBE.

3.1 Separating equilibria

In a separating PBE, first period prices signal the type of incumbent’s production cost and all information is revealed. Therefore, in the second period, when there is entry in market $A$, the entrant chooses his price under full information on the type of incumbent’s production cost, i.e., price competition in the second period occurs under complete information. The entrant achieves duopoly gross equilibrium profits given by, as mentioned, $\pi^E$ and $\pi^E$ when the incumbent has high and low production costs, respectively.

We assume that $\pi^E > f^E > \pi^E$, i.e., if the entrant believes that the incumbent has low production cost, the optimal decision is not to enter. Conversely, the entrant optimally decides to enter if he believes that the incumbent has high production cost. Therefore, second period incumbent’s profit are increasing in $\rho(p_A, p_B)$. If the posterior belief of the entrant is that the incumbent has low production cost, i.e., $\rho(p_A, p_B)$ equals 1, then there is no entry in market $A$ and the incumbent is a monopolist in the second period achieving $\pi_{Am}$. Since $\pi_{Am}$ is never smaller than $\pi_{Ad}$, both types of incumbent would like to convince the entrant to be low cost. However, the entrant is aware of this and only believes the incumbent to be low cost if he sends a credible signal.

The main idea in a separating PBE is that the low production cost incumbent may use costly signals $(p_{A}^*, p_{B}^*)$ to separate from his high production cost counterpart and thus deter entry. That is, the low cost incumbent may use first period prices $(p_{A}^*, p_{B}^*)$ that would be unattractive were the incumbent to have high production cost. The second period provides the reward needed to justify the low cost incumbent’s signaling cost. It follows that the low cost incumbent’s prices $(p_{A}^*, p_{B}^*)$ typically differ from the prices chosen under complete information. If incumbent’s production cost was known, in the first period the low cost incumbent would set the monopoly optimal price in each market, $(p_{Am}^*, p_{Bm}^*)$, i.e., the complete information prices.

The question addressed is whether there is a distortion of these prices when they are used to
signal incumbent’s production cost.

In a separating PBE, first period prices charged by the high and low cost incumbents differ, \((\bar{p}_A^*, \bar{p}_B^*) \neq (p_A^*, p_B^*)\). Thus, observing first period prices allows the entrant to become fully informed. If \((\bar{p}_A^*, \bar{p}_B^*)\) is observed, the posterior beliefs must be \(\rho(\bar{p}_A^*, \bar{p}_B^*) = 0\) whilst if \((p_A^*, \bar{p}_B^*)\) is observed, \(\rho(p_A^*, \bar{p}_B^*) = 1\). Off-the-equilibrium path beliefs are not restricted as Bayes’ rule is not applicable to events with zero probability.\(^8\) We assume that for all other \((p_A^*, \bar{p}_B^*)\), the posterior beliefs are \(\rho(p_A^*, \bar{p}_B^*) = 0\) (so that neither type of incumbent wants to deviate to \((p_A^*, \bar{p}_B^*)\) since the entrant always decides to enter unless when he observes \((p_A^*, \bar{p}_B^*)\)).

In any separating PBE, the high cost incumbent charges the optimal monopoly price in each market, \((p_A^*, \bar{p}_B^*) = (p_{A_{\text{m}}}, \bar{p}_{B_{\text{m}}})\). This can be shown by contradiction: if \((p_A^*, \bar{p}_B^*) \neq (p_{A_{\text{m}}}, \bar{p}_{B_{\text{m}}})\), the high cost incumbent would have an incentive to deviate to \((p_{A_{\text{m}}}, \bar{p}_{B_{\text{m}}})\) as this would increase first period profit and it would not decrease second period profit since the entrant always decides to enter unless when he observes \((p_A^*, \bar{p}_B^*)\).

On the other hand \((p_A^*, p_B^*)\) must be such that it is unprofitable for the high cost incumbent to mimic the low cost incumbent, \(i.e.:\)

\[
\pi_A(p_A^*) + \pi_B(p_B^*) + \delta (\bar{\pi}_{A_{\text{m}}} + \bar{\pi}_{B_{\text{m}}}) \leq \bar{\pi}_{A_{\text{m}}} + \bar{\pi}_{B_{\text{m}}} + \delta (\bar{\pi}_{A_{\text{d}}} + \bar{\pi}_{B_{\text{m}}}).
\]

Or equivalently:

\[
\delta (\bar{\pi}_{A_{\text{m}}} - \bar{\pi}_{A_{\text{d}}}) \leq (\bar{\pi}_{A_{\text{m}}} + \bar{\pi}_{B_{\text{m}}}) - (\left(\pi_A(p_A^*) + \pi_B(p_B^*)\right)). \quad (5)
\]

The left hand side of condition (5) is the second period gain if the high cost incumbent deviates from his equilibrium strategy and mimics the low cost incumbent. The right hand side of condition (5) is the first period loss if the high cost incumbent deviates from his equilibrium strategy. Let \(\tilde{C}\) be the set of \((p_A^*, \bar{p}_B^*)\) that satisfy the incentive compatibility condition (5). Figure 1 depicts the set \(\tilde{C}\) in light grey.

Moreover, for a separating PBE to exist, it must be true that \((p_A^*, \bar{p}_B^*)\) is optimal for the low

\(^8\)A perfect bayesian equilibrium requires Bayes’ consistency of beliefs. These posterior beliefs were obtained from the prior beliefs using Bayes’ rule and considering the incumbent’s equilibrium strategies. Since any other vector of prices besides \((p_A^*, \bar{p}_B^*)\) and \((p_A^*, \bar{p}_B^*)\) is charged with probability zero, Bayes’ rule cannot be applied for all other \((p_A^*, \bar{p}_B^*)\). Therefore, we may select any value for \(\rho(p_A^*, \bar{p}_B^*)\) for all other \((p_A^*, \bar{p}_B^*)\), \(i.e.,\) there is arbitrariness upon the off-the-equilibrium path posterior beliefs.
cost incumbent, i.e.:

\[ \pi_A (p_A^*) + \pi_B (p_B^*) + \delta (\pi_{Am} + \pi_{Bm}) \geq \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} + \pi_{Bm}) \cdot \]

Or equivalently:

\[ \delta (\pi_{Am} - \pi_{Ad}) \geq (\pi_{Am} + \pi_{Bm}) - (\pi_A (p_A^*) + \pi_B (p_B^*)) \cdot \]  \hfill (6)

Let \( \mathcal{C} \) be the set of \((p_A^*, p_B^*)\) that satisfy the incentive compatibility condition (6). Figure 2 depicts the set \( \mathcal{C} \) in grey.

The following proposition sets a sufficient condition for the existence of a separating PBE.

**Proposition 1** A separating PBE exists for \((p_A^*, p_B^*) = (\bar{p}_{Am}, \bar{p}_{Bm})\) and for some non-negative prices \((p_A^*, p_B^*)\) set by the low cost incumbent if and only if the incentive compatibility conditions (5) and (6) both hold, i.e., \((p_A^*, p_B^*) \in \mathcal{C} \cap \mathcal{C} \). Moreover if

\[ \pi_{Am} - \pi_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad} \]  \hfill (7)

then the set \( \mathcal{C} \cap \mathcal{C} \) is non-empty and consequently there exists a separating PBE.

**Proof:** To show necessity is enough to notice that if condition (5) or condition (6) fail, then either the high cost or the low cost incumbent is not behaving optimally in the proposed
separating PBE for any specification of the off-the-equilibrium beliefs. Thus one can not have a separating PBE. To show that conditions (5) and (6) are sufficient for a separating PBE to exist it is important to notice that in a separating PBE, by Bayes’ rule, $\rho(p_A^*, p_B^*) = 1$ and $\rho(p_A^*, p_B^*) = 0$. Assume that for all other $(p'_A, p'_B)$, $\rho(p'_A, p'_B)$ = 0. Given these beliefs and the fact that $\bar{\pi}^E > f^E > \pi^E$, it is optimal for the entrant not to enter when he observes $(p_A^*, p_B^*)$ and to enter otherwise. But then, given the entrant’s strategy, it is optimal for the high cost incumbent to set $(p_A^*, p_B^*)$ and the low cost incumbent to set $(\bar{p}_A^*, \bar{p}_B^*)$ if conditions (5) and (6) both hold. Notice that beliefs given by $\rho(p_A^*, p_B^*) = 1$, $\rho(p_A^*, p_B^*) = 0$ and for all other $(p'_A, p'_B)$, $\rho(p'_A, p'_B) = 0$ are consistent with the described equilibrium strategies. Therefore, pricing strategies $(p_A^*, p_B^*) = (\bar{p}_A^*, \bar{p}_B^*)$ and $(p_A^*, p_B^*) \in \mathcal{C} \cap \mathcal{C}$ support a separating PBE.

The existence of a separating PBE depends on whether $\mathcal{C} \cap \mathcal{C}$ is non-empty or not. Condition (7) means that it is more profitable to generate favorable beliefs (which deter entry) for the low cost incumbent than to the high cost incumbent: the low cost incumbent has larger financial capacity to incur the costly signaling prices. Condition (7) is the typical single-crossing condition which guarantees existence of a separating PBE. The proof that condition (7) is a sufficient condition for the existence of a separating PBE is shown in Appendix C.

Notice that if we assume that $\bar{\pi}_{Am} - \bar{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad}$, our set of parameters’ values
\{\tau, c^E, k\} becomes more restrict than \(M \cap D\). For the remaining analysis we consider \(k < 1\) and parameters’ values \(\{\tau, c^E, k\} \in M \cap \tilde{D}\) (see details of set \(\tilde{D}\) in Appendix D). For this set of parameters’ values, there are multiple separating perfect bayesian equilibria since the subset \(\mathfrak{C} \cap \mathfrak{C}\) is shown to be non-empty.

Figure 3 illustrates Proposition 1 where the subset \(\mathfrak{C} \cap \mathfrak{C}\), in dark grey, is the set of all possible separating perfect bayesian equilibria \((p_A^*, p_B^*)\).

The lack of restrictions upon off-the-equilibrium path posterior beliefs generates multiple separating perfect bayesian equilibria. The literature on refinements of perfect bayesian equilibria suggests plausibility criteria upon off-the-equilibrium path posterior beliefs. One of these criteria states that when the entrant observes an off-the-equilibrium path strategy, he should put probability zero on any type of incumbent for which the incumbent’s strategy is dominated, \(i.e.,\) the entrant is not allowed to believe that the incumbent sets a dominated strategy. If we impose the domination criterion, there emerges a unique separating PBE for each possible set of parameters’ values \(\{\tau, c^E, k, \delta\}\).

The pre-entry prices \((p_A, p_B)\) are considered to be dominated for an incumbent whose production cost is \(i, \ i\) standing for high or low, if:

\[
\pi^i_A(p_A) + \pi^i_B(p_B) + \delta(\pi^i_{Am} + \pi^i_{Bm}) < \pi^i_{Am} + \pi^i_{Bm} + \delta(\pi^i_{Ad} + \pi^i_{Bm}).
\]
Thus, pre-entry prices \((p_A, p_B)\) are \textit{dominated} for type \(i\) if this type of incumbent has less profit with these prices under the most favorable entry conditions (\textit{i.e.}, no entry) than with pre-entry monopoly prices in both markets under the least favorable entry conditions. If \((p_A, p_B)\) is dominated for a high cost incumbent but not dominated for a low cost incumbent, the entrant should not believe that the incumbent has high production cost when he observes \((p_A, p_B)\). Thus the only reasonable beliefs are \(\rho(p_A, p_B) = 1\).

The set of separating perfect bayesian \textit{equilibria}, \(C \cap C\), contains pre-entry prices \((p_A, p_B)\) which are \textit{dominated} for a high cost incumbent (since they satisfy condition (5) in inequality) but are not \textit{dominated} for a low cost incumbent (since they satisfy condition (6)). Therefore, the entrant’s reasonable beliefs are given by \(\rho(p_A, p_B) = 1\) for all \((p_A, p_B) \in C \cap C\) for which the incentive compatibility condition (5) is satisfied in inequality, \textit{i.e.}, for all \((p_A, p_B) \in C \cap C\) which are not in the frontier of \(C\) (we denote this set by \((\text{int}C) \cap C\)) since the entrant excludes the possibility that the high cost incumbent sets any of those pre-entry prices. Given these beliefs, the entrant will not enter if he observes any \((p_A, p_B)\) in \((\text{int}C) \cap C\) and he will also not enter if he observes the low cost incumbent’s \textit{equilibrium} prices \((p^*_A, p^*_B)\). The next lemma characterizes the low cost incumbent \textit{equilibrium} strategy under these beliefs.

\textbf{Lemma 1} The low cost incumbent strategy in any separating PBE immune to the domination criterion is the solution \((p^*_A, p^*_B)\) of the following problem:

\[
\max_{(p_A, p_B)} \pi_A p_A + \pi_B p_B
\]

subject to:

\[
\begin{align*}
\mathcal{IC}_L : & \pi_A (p_A) + \pi_B (p_B) \leq \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} - \pi_{Am}) \\
\mathcal{IC}_R : & \pi_A (p_A) + \pi_B (p_B) \geq \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} - \pi_{Am}),
\end{align*}
\]

where the two constraints are the incentive compatibility conditions (5) and (6), respectively. For a given combination of parameters’ values \(\{\delta, c^E, k, \delta\}\) there is a unique solution to this constrained optimization problem. Thus, there exists a unique separating PBE immune to the domination criterion.

\textbf{Proof}: If the solution of problem (8) is in \((\text{int}C) \cap C\), it is immediate that only this solution can survive the domination criterion. Since the entrant does not enter for any \((p_A, p_B)\)
(int\(\tilde{C}\)) \(\cap\) \(\mathbb{C}\), a low cost incumbent gets monopoly profit in the second period for any \((p_A, p_B)\) in \((int\tilde{C}) \cap \mathbb{C}\). But then for any \((p_A, p_B) \in \tilde{C} \cap \mathbb{C}\) and \((p_A, p_B) \neq (\bar{p}_A^*, \bar{p}_B^*)\) the low cost incumbent is not behaving optimally.

If the solution of problem (8) is in the frontier of \(\tilde{C}\) and we consider any \((p_A, p_B)\) in \(int\tilde{C}\) and \((p_A, p_B)\) in \(\mathbb{C}\), the low cost incumbent would always have an incentive to deviate to a pair of prices in \(int\tilde{C}\) closer to \((\bar{p}_A^*, \bar{p}_B^*)\). Thus, none of the separating equilibria in \((int\tilde{C}) \cap \mathbb{C}\) survives the domination criterion. Moreover, if we consider any \((p_A, p_B) \in \tilde{C} \cap \mathbb{C}\) and \((p_A, p_B)\) in the frontier of \(\tilde{C}\) but different from \((\bar{p}_A^*, \bar{p}_B^*)\) the low cost incumbent would gain by deviating to a pair of prices in \(int\tilde{C}\) arbitrarily close to \((\bar{p}_A^*, \bar{p}_B^*)\).

The uniqueness result is a direct consequence of solving the constrained optimization problem (8). □

Henceforth, we will use the expression «least cost separating equilibrium» to denote the unique separating PBE immune to the domination criterion. Let us now examine this equilibrium for each combination of parameters’ values \(\{\overline{c}, c^E, k, \delta\}\) where \(k < 1, \{\overline{c}, c^E, k\} \in \mathbb{M} \cap \tilde{D}\) and \(\delta \in ]0, 1]\). The next proposition characterizes the three types of least cost separating equilibria where \(IC\) is never binding.

**Proposition 2** The three types of least cost separating equilibria are as follows:

(i) When no constraint is binding and both \(p_A^*\) and \(p_B^*\) are positive:

\[
\begin{array}{c|c|c}
\text{Sol}_1 & 0 < \delta \leq \hat{\delta} & p_A^* = \overline{p}_A m \\
& & p_B^* = \overline{p}_B m
\end{array}
\]

(ii) When \(TC\) is the only binding constraint and both \(p_A^*\) and \(p_B^*\) are positive:

\[
\begin{array}{c|c|c}
\text{Sol}_2 & \hat{\delta} < \delta \leq \hat{\delta} & p_A^* = \overline{p}_A m - \frac{\delta (\pi_{Am} - \pi_{Ad}) b_A}{2} \\
& & p_B^* = \overline{p}_B m - \frac{\delta (\pi_{Am} - \pi_{Ad}) b_A}{2}
\end{array}
\]

(iii) When \(TC\) is the only binding constraint and only \(p_A^*\) is positive:

\[
\begin{array}{c|c|c}
\text{Sol}_3 & \hat{\delta} < \delta \leq 1 & p_A^* = \overline{p}_A m - \sqrt{\delta (\pi_{Am} - \pi_{Ad}) b_A} - \frac{(a_B + \overline{c})^2}{4}
\end{array}
\]

where:

\[
\hat{\delta} = \frac{\overline{c}^2}{2b_A (\pi_{Am} - \pi_{Ad})}
\]

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\[ \hat{\delta} = \frac{(a_B + v)^2}{2b_A (\pi_{Am} - \pi_{Ad})}. \]

**Proof:** The result is a direct consequence of solving the low cost incumbent’s optimization problem. Kuhn-Tucker conditions are described in Appendix E. ■

Notice that \( \hat{\delta} \) and \( \hat{\delta} \) are always positive and may be larger than 1 for some subset of parameters’ values \( \{\tau, c^E, k\} \in M \cap \hat{D} \) and \( k < 1 \). For cases where \( 1 \leq \hat{\delta} < \delta \), which happen for subset \( S_1 \) (see details in Appendix F), the solution for all \( k < 1 \) and \( \delta \in [0, 1] \) is given by \( \text{Sol}_1 \). At \( \text{Sol}_1 \), the least cost separating *equilibrium* equals the *complete information prices*, *i.e.*, the low cost incumbent pre-entry prices are the monopoly prices.

In addition, for cases where \( \hat{\delta} < 1 < \delta \), which happen for subset \( S_2 \) (see details in Appendix F), then for \( \delta \leq \hat{\delta} \), the solution is given by \( \text{Sol}_1 \) but for high \( \delta \), \( \delta > \hat{\delta} \), the solution is given by \( \text{Sol}_2 \). In the latter solution, there is a downward distortion in prices at the least cost separating *equilibrium* when \( \tau \) is positive since \( p_A^* < p_{Am} \) and \( p_B^* < p_{Bm} \).

Finally, if \( \hat{\delta} < 1 \), which happens for subset \( S_3 \) (see also details in Appendix F), then for:

(i) \( \delta \leq \hat{\delta} \), the solution is given by \( \text{Sol}_1 \);
(ii) \( \hat{\delta} < \delta \leq \delta \), the solution is given by \( \text{Sol}_2 \);
(iii) \( \delta > \hat{\delta} \), the solution is given by \( \text{Sol}_3 \).

At \( \text{Sol}_3 \), there is also a downward distortion in prices at the least cost separating *equilibrium* since \( p_A^* < p_{Am} \) and \( p_B^* = 0 < p_{Bm} \). Notice that both \( \text{Sol}_2 \) and \( \text{Sol}_3 \) present pre-entry prices lower than the *complete information prices*.

Therefore the least cost separating *equilibrium* may or may not exhibit a downward distortion in both pre-entry prices depending on parameters’ values \( \{\tau, c^E, k, \delta\} \). This conclusion poses a relevant question: what determines the degree of the downward distortion in prices when it occurs? The next subsection presents a characterization of the factors which influence the downward distortion in prices.

### 3.1.1 Discussion of results

At \( \text{Sol}_1 \), the low cost incumbent sets the monopoly prices at the least cost separating *equilibrium*. In this case, the low cost incumbent does not use costly signals, *i.e.*, there is no distortion associated with *efficient* signaling. This solution holds for \( \delta \leq \hat{\delta} \) where \( \hat{\delta} \) is increasing with
\( c \). Notice that the larger is \( c \), the larger is the difference between the two types of incumbent’s production cost. But if the two types of incumbent are very different, which implies very different monopoly prices, the low cost incumbent does not need to distort prices in order to separate himself from the high cost incumbent. In other words, when the two types of incumbent’s production cost are very different separation is easy.

For all other set of parameters’ values \( \{c, c^E, k, \delta\} \), the least cost separating equilibrium exhibits a downward distortion in both pre-entry prices. Most curiously, at \( Sol_2 \) the downward distortion of the price in market \( A \) is equal to the downward distortion of the price in market \( B \) even under our assumption of demand in market \( A \) being less elastic than demand in market \( B \) for a given market price. Simple computations reveal that the relative price decrease is always higher in market \( B \) since \( p_{Am} \) is always higher than \( p_{Bm} \).

Notice that there are several factors which influence the degree of this identical downward distortion in prices given by:

\[
\sqrt{\frac{\delta (\bar{\pi}_{Am} - \bar{\pi}_{Ad}) b_A}{2}}.
\]

The downward distortion in prices is increasing with \( \delta \). As \( \delta \) rises, the future becomes more important. Thus, if the high cost incumbent deviates and charges \( (p^*_A, p^*_B) \), his gain increases, which means that he has more incentive to mimic the low cost incumbent. But this implies that the low cost incumbent has to increase the distortion in prices in order to separate from his high cost counterpart. On the other hand, when \( c^E \) rises, \( i.e., \) when the entrant becomes less efficient, \( \bar{\pi}_{Ad} \) increases and thus the downward distortion in prices is smaller.

Interestingly, the degree of product substitutability also affects the level of the downward distortion in prices. When \( \tau \to 0 \), \( i.e., \) rival firms’ products are completely differentiated, then \( \bar{\pi}_{Am} \to \bar{\pi}_{Ad} \) and there is no downward distortion in prices. Conversely when \( \tau \to 1 \), \( i.e., \) rival firms’ products are highly substitutable, then \( \bar{\pi}_{Am} \) is much higher than \( \bar{\pi}_{Ad} \) and thus the downward distortion in prices is the highest possible. Therefore, when the degree of product substitutability rises, the downward distortion in prices is bigger. All these factors similarly determine the downward distortion of the price in market \( A \) at \( Sol_3 \) but there is an additional factor. When \( k \) decreases, \( i.e., \) demand in market \( A \) becomes much less elastic than demand in market \( B \) for a given market price, then the downward distortion of the price in market \( A \) is smaller.
3.2 Pooling Equilibria

We now analyze the pooling perfect bayesian equilibria. In a pooling PBE, first period prices of both types of incumbent are equal and thus do not signal the incumbent’s type. Therefore, the entrant does not change his prior beliefs when he observes the equilibrium pair of prices. We assume that, if entry occurs in market \( A \), the entrant learns the incumbent’s cost before price decisions are taken in the second period. Thus, price competition in the second period occurs under complete information.

As in subsection 3.1, we assume that \( \pi^E > f^E > \pi^E \). We further assume that \( \rho_o \pi^E + (1 - \rho_o) \pi_E < f^E \), which implies that under the prior beliefs the entrant’s optimal decision is not to enter. This condition is necessary for a pooling PBE to exist. In fact, if this condition failed and the two types charged the same prices, the entrant would enter when he observes the pooling pair of prices. Given this, each type of incumbent would have an incentive to charge his monopoly prices in the first period, thus the pooling strategy could not be optimal.

In a pooling PBE, first period prices charged by the high and low cost incumbents are the same, \((p^*_A, p^*_B) = (p^*_A, p^*_B)\). When \((p^*_A, p^*_B)\) is observed, by Bayes’ rule, the posterior beliefs are equal to the prior beliefs, i.e., \( \rho(p^*_A, p^*_B) = \rho_o \) and the entrant’s optimal decision is not to enter. We assume that for all other \((p'_A, p'_B)\) the posterior beliefs are \( \rho(p'_A, p'_B) = 0 \).

In any pooling PBE, \((p^*_A, p^*_B)\) must be optimal for the high cost incumbent, i.e.:

\[
\pi_A (p_A^*) + \pi_B (p_B^*) + \delta (\pi_{Am} + \pi_{Bm}) \geq \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} + \pi_{Bm}).
\]

Or equivalently:

\[
\delta (\pi_{Am} - \pi_{Ad}) \geq (\pi_{Am} + \pi_{Bm}) - (\pi_A (p_A^*) + \pi_B (p_B^*)). \tag{9}
\]

This condition is the reverse of condition (5) used in the analysis of separating PBE. Let \( C' \) be the set of \((p^*_A, p^*_B)\) that satisfy the incentive compatibility condition (9). Figure 1 depicts the set \( C' \) in white since in light grey we have the set of \((p'_A, p'_B)\) that satisfy condition (5).

Moreover, for a pooling PBE to exist, it must also be true that \((p^*_A, p^*_B)\) is optimal for the low cost incumbent, i.e.:

\[
\pi_A (p_A^*) + \pi_B (p_B^*) + \delta (\pi_{Am} + \pi_{Bm}) \geq \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} + \pi_{Bm}).
\]

Or equivalently:

\[
\delta (\pi_{Am} - \pi_{Ad}) \geq (\pi_{Am} + \pi_{Bm}) - (\pi_A (p_A^*) + \pi_B (p_B^*)). \tag{10}
\]
This is precisely the same condition as (6). Therefore, Figure 2 depicts in grey the set of \((p_A^*, p_B^*)\) that satisfy the condition (10).

The following proposition summarizes the conditions for a pooling PBE.

**Proposition 3** A pooling PBE exists for some non-negative prices \((p_A^*, p_B^*)\) set by the high and low cost incumbents if and only if the incentive compatibility conditions (9) and (10) both hold, i.e., \((p_A^*, p_B^*) \in \bar{C}' \cap C\).

**Proof:** To show necessity is enough to notice that if either condition (9) or (10) fail, then either the high cost or the low cost incumbent is not behaving optimally in the proposed pooling PBE for any specification of the off-the-equilibrium beliefs, thus one can not have a pooling PBE. To show that conditions (9) and (10) are sufficient for a pooling PBE to exist it is important to notice that in a pooling PBE, by Bayes’ rule, \(\rho(p_A^*, p_B^*) = \rho_o\). Assume that for all other \((p_A', p_B')\), \(\rho(p_A', p_B') = 0\). Given these beliefs and the assumptions over the entrant’s profits, it is optimal for the entrant not to enter when he observes \((p_A^*, p_B^*)\) and to enter otherwise. But then, given the entrant’s strategy, it is optimal for the high and low cost incumbents to set \((p_A^*, p_B^*)\) if conditions (9) and (10) both hold. Notice that beliefs given by \(\rho(p_A^*, p_B^*) = \rho_o\) and for all other \((p_A', p_B')\), \(\rho(p_A', p_B') = 0\) are consistent with the described equilibrium strategies. Therefore, pricing strategies \((\tilde{p}_A^*, \tilde{p}_B^*) = (p_A^*, p_B^*)\) and \((p_A^*, p_B^*) \in \bar{C}' \cap C\) support a pooling PBE. ■

Under the assumption that \(\bar{\pi}_A \geq \bar{\pi}_A > \bar{\pi}_A - \bar{\pi}_A, \bar{C}' \cap C\) is shown to be non-empty as long as the two types of incumbent are not too different (\(C\) is not too high). Thus, for this set of parameters’ values, there are multiple pooling perfect bayesian equilibria. The lack of restrictions upon off-the-equilibrium path posterior beliefs generates multiple pooling perfect bayesian equilibria. The use of equilibrium refinements allows us to eliminate several perfect bayesian equilibria. The existence of pooling equilibria which survive the domination criterion depends on whether \((p_{Am}, p_{Bm})\) belongs or does not belong to \(\bar{C}' \cap C\). Next we analyze these two cases.

### 3.2.1 When \((p_{Am}, p_{Bm}) \notin \bar{C}' \cap C\)

Figure 4 illustrates an example where \((p_{Am}, p_{Bm}) \notin \bar{C}' \cap C\). The set of \((p_A^*, p_B^*) \in \bar{C}' \cap C\), in light grey, is the set of pooling perfect bayesian equilibria.
In this case it is easy to show that no pooling PBE survives the domination criterion. Since any pre-entry pair of prices \((p_A, p_B)\) in \((\text{int}\tilde{C}) \cap C\) is strictly dominated for a high cost incumbent but is not dominated for a low cost incumbent, then \(\rho(p_A, p_B) = 1\) for such pair of prices. Given these beliefs, the entrant will not enter if he observes any \((p_A, p_B)\) in \((\text{int}\tilde{C}) \cap C\). Therefore, in any \((p_A^*, p_B^*) \in \tilde{C} \cap C\), the low cost incumbent would always have an incentive to deviate to \((p_{Am}, p_{Bm})\) since first period profit is higher and entry does not occur.

3.2.2 When \((p_{Am}, p_{Bm}) \in \tilde{C} \cap C\)

Figure 5 illustrates an example where \((p_{Am}, p_{Bm}) \in \tilde{C} \cap C\). The set of \((p_A^*, p_B^*) \in \tilde{C} \cap C\), in grey, is the set of pooling perfect bayesian equilibria.

When \((p_{Am}, p_{Bm}) \in \tilde{C} \cap C\), some pooling equilibria are immune to the domination criterion. Let \(\overline{\pi}_{A,B}\) be the level of profit where the low cost incumbent’s isoprofit is tangent to the frontier of the incentive compatibility condition (9). All the pooling perfect bayesian equilibria where pre-entry profits of the low cost incumbent are lower than \(\overline{\pi}_{A,B}\) (outside the area in squares) do not survive the domination criterion. Consider any such pooling PBE. Since the entrant will not enter if he observes any \((p_A, p_B)\) in \((\text{int}\tilde{C}) \cap C\) once \(\rho(p_A, p_B) = 1\), the low cost incumbent would always have an incentive to deviate from the proposed pooling PBE to a pair of prices in
Figure 5: Subset $\bar{C} \cap C$ in grey

$(int \bar{C}) \cap C$ arbitrarily close to $(p^*_A, p^*_B)$. Therefore, the subset of pooling PBE in $\bar{C} \cap C$ that are immune to the domination criterion is given by all $(p^*_A, p^*_B)$ where pre-entry profits of the low cost incumbent are higher than $\pi_{A,B}$. We denote this subset by $I \subset \bar{C} \cap C$. Figure 5 depicts this subset in the area marked with squares.

If we further refine the equilibrium concept using Cho and Kreps (1987) intuitive criterion we can eliminate some more pooling perfect bayesian equilibria in subset $I$. Cho and Kreps criterion states that a type should not be expected to play an equilibrium dominated action. An action $a$ can be eliminated for type $\theta$ by equilibrium dominance if type $\theta$ does not gain by deviating from the equilibrium no matter what action the uninformed player chooses. Thus, if the uninformed player observes action $a$ he should put zero probability on type $\theta$. This reasonability criterion upon off-the-equilibrium path posterior beliefs may change the uninformed player’s best response, which in turn may induce some other type to deviate from the proposed equilibrium. If this happens, the equilibrium does not survive the intuitive criterion. To understand better this criterion, let us consider some pooling PBE, $(p^*_A, p^*_B) \in I$. Suppose that there exists a deviant pair of prices $(\tilde{p}_A, \tilde{p}_B)$ such that a high cost incumbent would not want to deviate from $(p^*_A, p^*_B)$ even if he was believed to be low cost by charging $(\tilde{p}_A, \tilde{p}_B)$ but, given these beliefs, the low cost incumbent’s profit would be higher with $(\tilde{p}_A, \tilde{p}_B)$ than his equilibrium profit. That
is, \((\tilde{p}_A, \tilde{p}_B)\) is an equilibrium dominated action for the high cost incumbent but not for the low cost incumbent, thus \(\rho(\tilde{p}_A, \tilde{p}_B) = 1\) is the appropriate inference if \((\tilde{p}_A, \tilde{p}_B)\) is observed. Since the entrant will not enter if he observes \((\tilde{p}_A, \tilde{p}_B)\) the low cost incumbent gains by deviating from \((p^*_A, p^*_B)\) and the equilibrium fails the intuitive criterion. In order for \((\tilde{p}_A, \tilde{p}_B)\) to exist the following two conditions have to hold:

\[
\bar{\pi}_A (\tilde{p}_A) + \bar{\pi}_B (\tilde{p}_B) + \delta (\bar{\pi}_Am + \bar{\pi}_Bm) > \bar{\pi}_A (p^*_A) + \bar{\pi}_B (p^*_B) + \delta (\bar{\pi}_Am + \bar{\pi}_Bm) \Leftrightarrow
\]

\[
\Leftrightarrow \bar{\pi}_A (\tilde{p}_A) + \bar{\pi}_B (\tilde{p}_B) > \bar{\pi}_A (p^*_A) + \bar{\pi}_B (p^*_B)
\]

and

\[
\bar{\pi}_A (\tilde{p}_A) + \bar{\pi}_B (\tilde{p}_B) + \delta (\bar{\pi}_Am + \bar{\pi}_Bm) < \bar{\pi}_A (p^*_A) + \bar{\pi}_B (p^*_B) + \delta (\bar{\pi}_Am + \bar{\pi}_Bm) \Leftrightarrow
\]

\[
\Leftrightarrow \bar{\pi}_A (\tilde{p}_A) + \bar{\pi}_B (\tilde{p}_B) < \bar{\pi}_A (p^*_A) + \bar{\pi}_B (p^*_B).
\]

Notice that these conditions represent the set of prices where pre-entry profits of the low cost incumbent are higher than his pooling PBE pre-entry profit and where pre-entry profits of the high cost incumbent are lower than his pooling PBE pre-entry profit, respectively.

Using the intuitive criterion we eliminate most pooling perfect bayesian equilibria. Any pooling PBE above the isoprofit curve of the high cost incumbent with profit level \(\bar{\pi}(p_{Am}, p_{Bm}) = \bar{\pi}_A (p_{Am}) + \bar{\pi}_B (p_{Bm})\) fails the intuitive criterion, since \((p_{Am}, p_{Bm})\) is equilibrium dominated for the high cost incumbent and it is a profitable deviation for the low cost incumbent. Moreover, any pooling PBE with \(\bar{\pi}(p^*_A, p^*_B) \leq \bar{\pi}(p_{Am}, p_{Bm})\) where the isoprofit curves of the two types intercept can not survive the intuitive criterion as one can find pairs of prices which are below the high cost isoprofit curve but above the low cost isoprofit curve. Figure 6 illustrates a pair of prices, \((\hat{p}^*_A, \hat{p}^*_B)\), where the isoprofit curves (in grey) intercept. The area marked with hexagons corresponds to the pairs of prices which are equilibrium dominated for the high cost incumbent and where the low cost incumbent gains by deviating from \((\hat{p}^*_A, \hat{p}^*_B)\). Thus \((\hat{p}^*_A, \hat{p}^*_B)\) fails the intuitive criterion.

However, the pooling equilibria where the two isoprofit curves are tangent survive the intuitive criterion. Since the low cost isoprofit curve is steeper than the high cost isoprofit curve, any \((\tilde{p}_A, \tilde{p}_B)\) below the high cost isoprofit curve is necessarily below the low cost isoprofit curve, hence one can not find a \((\tilde{p}_A, \tilde{p}_B)\) which is equilibrium dominated for the high cost incumbent.
Figure 6: A pooling PBE that fails the intuitive criterion

but not for the low cost incumbent. The set of pooling equilibria where the isoprofits are tangent and thus survive the intuitive criterion is the line-segment from \((p_A^*, p_B^*)\) to \((p_{Am}, p_{Bm})\) (see Figure 7). It is interesting to notice that the pooling equilibria also entail a downward distortion with equal distortions in both markets.

### 3.3 Summary and further equilibrium refinements

At this point it is useful to summarize the equilibria results. Assuming \(\pi_{Am} - \pi_{Ad} > \tilde{\pi}_{Am} - \tilde{\pi}_{Ad}\) and \(\tilde{\pi}^E > f^E > \pi^E\), there always exists a unique separating PBE which is immune to the domination criterion. If, in addition, \(\rho_o\pi^E + (1 - \rho_o) \tilde{\pi}^E > f^E\), there are no pooling equilibria. Therefore, there exists a unique reasonable PBE which is the least cost separating equilibrium characterized in Proposition 2.

On the other hand, if \(\rho_o\pi^E + (1 - \rho_o) \tilde{\pi}^E < f^E\) we also have a set of pooling equilibria as long as the two types of incumbent are not too different (\(\bar{c}\) is not too high). In this case, if \((p_{Am}, p_{Bm})\) is a dominated strategy for the high cost incumbent, none of the pooling equilibria survives the domination criterion and the unique reasonable PBE is the least cost separating equilibrium where each type charges the monopoly prices in the first period.
Finally, when \( \rho_o \pi^E + (1 - \rho_o) \bar{\pi}^E < f^E \) and the two types of incumbent are so similar that \((\underline{p}_{Am}, \underline{p}_{Bm})\) can be supported as a pooling PBE, the set of pooling equilibria in the line-segment from \((p^*_A, p^*_B)\) to \((\underline{p}_{Am}, \underline{p}_{Bm})\) in Figure 7 survive the intuitive criterion. In this case, we have multiple perfect bayesian equilibria which survive the intuitive criterion: the least cost separating equilibrium and the set of pooling equilibria in the line-segment from \((\underline{p}_{Am}, \underline{p}_{Bm})\) to \((\underline{p}_{Am}, \underline{p}_{Bm})\). However, one can impose further refinements upon off-the-equilibrium path beliefs using Grossman and Perry (1986) criterion. According to this criterion if there exists a set \( K \) of types who are better of choosing action \( a \) than the equilibrium strategy while the remaining types are worse of, given that the uninformed player revises his beliefs to believe that \( t \in K \) and according to Bayes’ rule, then the proposed equilibrium is overturned. The next result shows that there exists a unique PBE which survives the Grossman and Perry (1986) criterion.

**Proposition 4** Suppose that \( \underline{\pi}_{Am} - \underline{\pi}_{Ad} > \bar{\pi}_{Am} - \bar{\pi}_{Ad} \) and \( \bar{\pi}^E > f^E > \underline{\pi}^E \) then there exists a unique PBE which survives the Grossman and Perry criterion which is equal to:

(i) the least cost separating equilibrium characterized in Proposition 2, when \( \rho_o \bar{\pi}^E + (1 - \rho_o) \bar{\pi}^E > f^E \).

(ii) the least cost separating equilibrium where \((\underline{p}_{Am}, \underline{p}_{Bm}) = (\underline{p}_{Am}, \underline{p}_{Bm})\), when \( \rho_o \bar{\pi}^E + (1 - \rho_o) \bar{\pi}^E < f^E \) and \((\underline{p}_{Am}, \underline{p}_{Bm})\) is a dominated strategy for the high cost incumbent.

(iii) the pooling equilibrium with \((\underline{p}_{Am}, \underline{p}_{Bm}) = (\underline{p}_{Am}, \underline{p}_{Bm})\), when \( \rho_o \bar{\pi}^E + (1 - \rho_o) \bar{\pi}^E < f^E \) and \((\underline{p}_{Am}, \underline{p}_{Bm})\) is not a dominated strategy for the high cost incumbent.
Proof: In case (i) and (ii) there exists a unique PBE which survives the domination criterion and consequently the intuitive criterion. Since Grossman and Perry criterion is a refinement of the intuitive criterion, if it exists a PBE which survives the Grossman and Perry criterion, it is necessarily the least cost separating equilibrium. In each case, it is also easy to prove that the least cost separating equilibrium is immune to Grossman and Perry criterion.

In case (iii), the only PBE which is immune to Grossman and Perry criterion is the pooling PBE where both types choose first period prices \((p_{Am}, p_{Bm})\). Let us first eliminate the remaining pooling equilibria. Since Grossman and Perry criterion is a refinement of the intuitive criterion, we just need to show that no other vector of prices in the line-segment from \((p^*_A, p^*_B)\) to \((p_{Am}, p_{Bm})\) in Figure 7 survives the criterion. Consider any pooling PBE, \((p^*_A, p^*_B)\), in the line segment between \((p^*_A, p^*_B)\) and \((p_{Am}, p_{Bm})\) such that \((p^*_A, p^*_B) < (p_{Am}, p_{Bm})\). Both types of incumbent would gain by deviating to \((p_{Am}, p_{Bm})\) if the entrant beliefs are such that he does not enter. Hence the entrant’s posterior beliefs following \((p_{Am}, p_{Bm})\) should be equal to the prior beliefs, \(\rho_o\), and his optimal decision is not to enter. Since there are consistent beliefs such that both types of incumbent gain by deviating, the proposed equilibrium fails the Grossman and Perry criterion. The least cost separating equilibrium, in which the low cost incumbent’s prices are \((p^*_A, p^*_B)\), can also be eliminated using the same reasoning. Both types would prefer to deviate from their separating equilibrium strategies to \((p_{Am}, p_{Bm})\) if the entrant beliefs are such that he does not enter. Then beliefs following \((p_{Am}, p_{Bm})\) should be \(\rho_o\), which implies that both types of incumbent gain by deviating from the least cost separating equilibrium.

It is interesting to notice that the unique PBE which survives the Grossman and Perry criterion is also the unique Pareto optimal equilibrium for the incumbent’s types. Moreover, this unique PBE entails a downward distortion for at least one type, except in case (i) when \(\delta \leq \hat{\delta}\) and in case (ii).

4 Extension of results

In this section, we analyze the implications of changing two of the initial assumptions on results. Firstly, we explore what happens if \(k \geq 1\), i.e., demand in market \(B\) is equally elastic or less
elastic than demand in market \( A \) for a given market price.\(^9\) Secondly, we extend the initial assumptions on the relationships between market \( A \) and market \( B \)’s parameters by considering that \( b_B = \frac{b_A}{w} \). Our initial assumption is a particular case of this assumption since \( b_B = \frac{b_A}{w} \) equals (4) when \( w = 1 \).

For \( k \geq 1 \), the set of parameters’ values where monopoly and duopoly quantities are non-negative and where condition (7) holds is given by \( \{c, c^E, k\} \in \tilde{D} \) once \( \tilde{D} \) is sufficient to guarantee that non-negative constraints are satisfied. For this set of parameters’ values, there may occur two types of least cost separating \textit{equilibrium}: \( \text{Sol}_1 \) and \( \text{Sol}_2 \). \( \text{Sol}_3 \) is impossible to occur since market \( B \) is now larger. Notice that \( \text{Sol}_1 \) and \( \text{Sol}_2 \) are both set, for different levels of \( \delta \), at the least cost separating \textit{equilibrium} for a subset of \( \tilde{D} \) where \( c^E < c^E \). Conversely, when \( c^E \geq c^E \), only \( \text{Sol}_1 \) is set at the least cost separating \textit{equilibrium} for all \( \delta \). For \( k > 1 \), the relative downward distortion in price is always higher in market \( A \) since \( p_{Am} \) is always lower than \( p_{Bm} \). When \( k = 1 \), the relative downward distortion in price is identical in both markets. The unique reasonable pooling PBE is \( (p_{Am}, p_{Bm}) \) where for \( k \geq 1 \), \( p_{Bm} \geq p_{Am} \).

Lets explore the impact of the extended assumption over the relationships between market \( A \) and market \( B \)’s parameters, \( b_B = \frac{b_A}{w} \), but considering \( k < 1 \). Notice that this new assumption does not influence the relationship between markets’ demand elasticity since given the quadratic utility function (2) and the resultant demand functions, demand in market \( A \) is less elastic than demand in market \( B \) for a given market price when \( k < 1 \). It certainly influences the relationship between consumer’s marginal utility in market \( B \) and consumer’s marginal utility in market \( A \). Thus, we now face several different scenarios where the monopoly demand functions in market \( A \) and market \( B \) do not have identical slopes. Therefore, for the set of parameters’ values \( \{c, c^E, k\} \in M \cap \tilde{D} \) and \( k < 1 \), there is impact upon two of the three types of the least cost separating \textit{equilibrium} that may occur. More precisely upon the solutions where there is a downward distortion in prices. The downward distortion in price continues to be identical in both markets (since the Kuhn-Tucker conditions are similar) but this decrease in prices now depends on \( w \). At \( \text{Sol}_2 \) the identical downward distortion in price is now given by \( \sqrt{\frac{\beta(p_{Am} - p_{Ad})b_A}{(1+w)}} \). If

---

\(^9\)This is the set of parameters that may capture a predatory dumping behavior. If we consider market \( B \) as the domestic market and market \( A \) as the external market, the incumbent may set a lower pre-entry price in the external market so as to deter entry. But notice that the incumbent uses prices in both markets as \textit{predatory signals}. 

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$w > 1$, there is a smaller downward distortion in price than when $w = 1$. Moreover as $w$ rises, the downward distortion in prices becomes smaller. Also at Sol$_3$, the downward distortion in price is now given by $\sqrt{\delta (\bar{\pi}_{Am} - \bar{\pi}_{Ad})} b_{A} - \frac{w(a_{B} + \bar{\sigma})^{2}}{4}$. There is also an impact on these solutions’ limits since $\delta$ now equals $\frac{(1+w)^{2}}{4b_{A}(\bar{\pi}_{Am} - \bar{\pi}_{Ad})}$ and $\bar{\delta}$ equals $\frac{(1+w)(a_{B} + \bar{\sigma})^{2}}{4b_{A}(\bar{\pi}_{Am} - \bar{\pi}_{Ad})}$ and thus the subsets $S_{1}$, $S_{2}$ and $S_{3}$ are different.

5 Conclusion

This paper explores the incumbent’s choice of a multiple signaling strategy when practicing discriminatory limit pricing. Considering potential entry in one of the two geographical independent markets where the incumbent operates, the incumbent uses third-degree price discrimination to deter entry when the entrant has incomplete knowledge about the incumbent’s production cost. The entrant offers a differentiated product and decides whether to enter after observing incumbent’s pre-entry prices in both markets. Thus, the incumbent may have an incentive to distort both pre-entry prices in order to convey information on production cost to the entrant so as to deter entry.

In the presence of this signaling effect we show that incumbent’s complete information prices may no longer hold. We show that there is always a unique reasonable perfect bayesian equilibrium. There are some cases where the only equilibrium that survives the domination criterion is the least cost separating equilibrium. In this separating equilibrium, the low cost incumbent’s pre-entry prices may have a downward distortion with respect to the complete information setting. The low cost incumbent uses pre-entry prices in both markets to signal low production cost. Moreover, we show that this distortion is identical in both markets for every relationship between markets’ demand parameters. The price distortion is increasing with the discount factor, the degree of product substitutability and the efficiency of the entrant. For the remaining cases, we show that the unique equilibrium that survives the Grossman and Perry (1986) criterion is a pooling equilibrium where both types of incumbents set the low cost incumbent’s monopoly prices.

We extend previous analysis of entry deterrence literature to a setup which considers third-degree price discrimination and product differentiation. In a multimarket setup, the incumbent uses multiple signaling. An implication of our analysis to predatory pricing detection is that
policy regulation authorities should reach worldwide information since firms are becoming global and may price lower than the monopoly prices in more than one market in order to deter entry. This paper shows that in some cases multinational firms may not be using purely discriminatory pricing but *discriminatory limit pricing*.

Further work should include the presence of an active regulator, for instance, in only one of the incumbent’s operating markets. This may lead to the use of signaling exclusively in the unregulated market. Three other extensions seem worth considering. The analysis of other incumbent’s strategies, such as accommodation strategies, which may lead to an upward distortion in prices. It would also be interesting to relax the assumption of constant marginal production cost. Finally, oligopolist incumbent firms would also be a relevant framework to consider since two-sided signaling arises.
A  Firms’ profit functions

Due to the assumption of constant marginal production cost, the incumbent maximizes profits in each market independently when using third-degree price discrimination.

A.1 Monopoly in market A and B

The incumbent’s monopoly profit in market $i$, $i = A, B$, when the incumbent has high production cost is given by:

$$\pi_i = (p_i - c) \left( \frac{a_i}{b_i} - \frac{1}{b_i} p_i \right).$$

The unique maximizer of this profit function is the monopoly discriminatory price in market $i$:

$$p_{im} = \frac{a_i + c}{2}$$

and the respective monopoly discriminatory quantity and profit in market $i$ are given by:

$$q_{im} = \frac{a_i - c}{2b_i} \text{ and } \pi_{im} = \frac{(a_i - c)^2}{4b_i}.$$

Analogously, when the incumbent has low production cost and $c = 0$, the monopoly discriminatory price, quantity and profit in market $i$ simplify to:

$$p_{im} = \frac{a_i}{2}, q_{im} = \frac{a_i}{2b_i} \text{ and } \pi_{im} = \frac{a_i^2}{4b_i}.$$

A.2 Duopoly in market A

When entry occurs in market $A$, firms’ profit functions when the incumbent has high production cost are given by:

$$\pi_{Ad} = (p_{Ad} - c) \left( \alpha - \beta p_{Ad} + \gamma p^E \right)$$

$$\pi^E = (p^E - c^E) \left( \alpha - \beta p^E + \gamma p_{Ad} \right).$$

As firms choose their prices simultaneously and independently, firms maximize their profit functions. The second period Nash equilibrium prices and quantities are:

$$p_{Ad} = \frac{\alpha (2\beta + \gamma) + \beta \gamma c^E + 2\beta^2 c}{4\beta^2 - \gamma^2}$$

$$p^E = \frac{\alpha (2\beta + \gamma) + 2\beta^2 c^E + \beta \gamma c}{4\beta^2 - \gamma^2}.$$
\[ \bar{q}_{Ad} = \frac{\beta (\alpha (2\beta + \gamma) + \beta \gamma c^E - (2\beta^2 - \gamma^2) \bar{c})}{4\beta^2 - \gamma^2} \]
\[ \bar{q}^E = \frac{\beta (\alpha (2\beta + \gamma) + \beta \gamma^2 - (2\beta^2 - \gamma^2) c^E)}{4\beta^2 - \gamma^2} \]

and the firms’ profits are given by:
\[ \bar{\pi}_{Ad} = \frac{\beta [\alpha (2\beta + \gamma) + \beta \gamma c^E - (2\beta^2 - \gamma^2) \bar{c}]^2}{(4\beta^2 - \gamma^2)^2} \]
\[ \bar{\pi}^E = \frac{\beta [\alpha (2\beta + \gamma) + \beta \gamma^2 - (2\beta^2 - \gamma^2) c^E]^2}{(4\beta^2 - \gamma^2)^2}. \]

Similarly, when the incumbent has low production cost and as \( c = 0 \), the second period equilibrium prices, quantities and profits in market \( A \) simplify to:
\[ p_{Ad} = \frac{\alpha (2\beta + \gamma) + \beta \gamma c^E}{4\beta^2 - \gamma^2} \]
\[ p^E = \frac{\alpha (2\beta + \gamma) + 2\beta^2 c^E}{4\beta^2 - \gamma^2} \]
\[ q_{Ad} = \frac{\beta (\alpha (2\beta + \gamma) + \beta \gamma c^E)}{4\beta^2 - \gamma^2} \]
\[ q^E = \frac{\beta (\alpha (2\beta + \gamma) - (2\beta^2 - \gamma^2) c^E)}{4\beta^2 - \gamma^2} \]

and:
\[ \bar{\pi}_{Ad} = \frac{\beta [\alpha (2\beta + \gamma) + \beta \gamma c^E]^2}{(4\beta^2 - \gamma^2)^2} \]
\[ \bar{\pi}^E = \frac{\beta [\alpha (2\beta + \gamma) - (2\beta^2 - \gamma^2) c^E]^2}{(4\beta^2 - \gamma^2)^2}. \]

### B Non-negative constraints analysis

From the monopoly analysis in Appendix A.1, if we assume (3) and (4) monopoly price, quantity and profit in market \( B \) for high and low production costs can be rewritten as:
\[ p_{Bm} = \frac{a_B + \bar{c}}{2} = \frac{ka_A + \bar{c}}{2}, \bar{q}_{Bm} = \frac{a_B - \bar{c}}{2b_B} = \frac{ka_A - \bar{c}}{2b_A} \]
\[ \bar{\pi}_{Bm} = \frac{(a_B - \bar{c})^2}{4b_B} = \frac{(ka_A - \bar{c})^2}{4b_A} \]
\[ \frac{\mathcal{P}_{Bm}}{2} = \frac{ka_A}{2}, \quad \frac{\mathcal{P}_{Am}}{2b_B} = \frac{ka_A}{2b_A} \quad \text{and} \quad \frac{\pi_{Bm}}{4b_B} = \frac{ka_A^2}{4b_A}. \]

When \( k < 1 \), market \( A \) and market \( B \) monopoly quantities are non-negative if \( \bar{\sigma} \leq ka_A \). Let \( \mathbf{M} \) be the set of parameters’ values \( \bar{\sigma} \) and \( k \) such that \( \bar{\sigma} \leq ka_A \). Notice that if \( k < 1 \), monopoly discriminatory price is higher in market \( A \). This result is expected: a discriminatory monopolist charges a higher price in the less elastic market. When \( k < 1 \), demand in market \( A \) is larger and less elastic than demand in market \( B \). Thus \( p_{Am}^I > p_{Bm}^I \) for both types of incumbent’s production cost.

If we focus on the duopoly analysis in market \( A \), presented in Appendix A.2, from non-negativity constraints of duopoly equilibrium quantities it follows that, for different values of \( \bar{\sigma} \), the set of possible \( c^E \) changes. We use \( \tau = \frac{d_A^*}{a_A^*} \), where \( \tau \) is a measure of the degree of product substitutability, in the following conditions. Therefore, if:

\[
\frac{a_A(1-\tau)(4+\tau)}{(2-\tau^2)} \leq \bar{\sigma} \leq \frac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} \quad \text{then} \quad c^E \in \left[ 0, \frac{a_A(1-\tau)(2+\tau)}{(2-\tau^2)} \right] \]

We consider \( \mathbf{D} \) to be this set of parameters’ values \( \bar{\sigma} \) and \( c^E \) where duopoly equilibrium quantities are non-negative.

C Proof of Proposition 1

When \( (\mathcal{P}_{Am}, \mathcal{P}_{Bm}) \in \mathbf{C} \), the proof is immediate since \( (\mathcal{P}_{Am}, \mathcal{P}_{Bm}) \in \mathbf{C} \) and thus \( \mathbf{D} \cap \mathbf{C} \) is non-empty. Suppose then that \( (\mathcal{P}_{Am}, \mathcal{P}_{Bm}) \notin \mathbf{C} \). Let \( (p_A', p_B') \) be a pair of prices in the frontier of the incentive compatibility condition of the high cost incumbent given by condition (5), i.e.:

\[
\pi_A (p_A') + \pi_B (p_B') + \delta \pi_{Am} = \pi_{Am} + \pi_{Bm} + \delta \pi_{Ad} \iff
\]

\[
\pi_A (p_A') + \pi_B (p_B') - \pi_{Am} - \pi_{Bm} + \delta (\pi_{Am} - \pi_{Ad}) = 0.
\]

Let \( p_A' < \mathcal{P}_{Am} \) and \( p_B' < \mathcal{P}_{Bm} \). Notice that:

\[
\pi_A (p_A') + \pi_B (p_B') - \pi_{Am} - \pi_{Bm} + \delta (\pi_{Am} - \pi_{Ad}) = \pi_A (p_A') + \pi_B (p_B') - \pi_{Am} - \pi_{Bm} + \delta (\pi_{Am} - \pi_{Ad}) - \left[ \pi_A (p_A') + \pi_B (p_B') - \pi_{Am} - \pi_{Bm} + \delta (\pi_{Am} - \pi_{Ad}) \right]
\]

\[
= \pi_A (p_A') - \pi_A (p_A') + \pi_B (p_B') - \pi_{Bm} + \delta [ (\pi_{Am} - \pi_{Ad}) - (\pi_{Am} - \pi_{Ad}) ]
\]

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As \( p'_A < \overline{p}_{Am} \) and \( p'_B < \overline{p}_{Bm} \), then this last expression is positive. Therefore, if we assume condition (7), there is always some \((p'_A, p'_B)\) where:

\[
\pi_A (p'_A) + \pi_B (p'_B) - \pi_{Am} - \pi_{Bm} + \delta (\pi_{Am} - \pi_{Ad}) > 0.
\]

This means that \((p'_A, p'_B)\) is in the interior of \( C \) and then \( \bar{C} \cap C \) is non-empty.

Therefore, condition (7) is a sufficient condition for the existence of a separating PBE.

### D Single-crossing condition analysis

Assuming \( \pi_{Am} - \pi_{Ad} > \pi_{Am} - \pi_{Ad} \) is equivalent to \( c^E < \tilde{c}^E \) where:

\[
\tilde{c}^E = \frac{\tau (\tau^4 - 5\tau^2 + 8) \bar{c} + 2a_A (1 - \tau) (2 + \tau) (\tau^3 - \tau^2 - 2\tau + 4)}{8 (2 - \tau^2)}, \quad \text{using } \tau = \frac{d_A}{b_A}.
\]

This single-crossing condition has impact on possible parameter’s values of \( c^E \). Thus, let \( \tilde{D} \) be the following set of parameters’ values \( \bar{c} \) and \( c^E \) where if:

\[
\begin{align*}
\frac{a_A (1 - \tau) (2 + \tau)}{(2 - \tau^2)} &\leq \bar{c} \quad \text{then } c^E \in \left[ 0, \frac{\bar{c}^E}{\tau} \right] \\
\frac{2a_A (1 - \tau) (4 - \tau^2)}{(2 - \tau^2)} &< \bar{c} \leq \frac{a_A (1 - \tau) (4 - \tau^2)}{(2 - \tau^2)} \quad \text{then } c^E \in \left[ \frac{(2 - \tau^2) \bar{c} - a_A (1 - \tau) (2 + \tau)}{\tau}, \frac{a_A (1 - \tau) (2 + \tau)}{(2 - \tau^2)} \right].
\end{align*}
\]

Therefore, adding non-negative constraints and single-crossing condition implies restricting the analysis to the parameters’ values \( \{\bar{c}, c^E, k\} \in M \cap \tilde{D} \).

### E Low cost incumbent firm’s Kuhn-Tucker conditions

Let \( \lambda \) and \( \mu \) be the Lagrange multipliers associated with \( TC \) and \( LC \), respectively. We can construct the Lagrangian function:

\[
\mathcal{L} = \pi_A (p^*_A) + \pi_B (p^*_B) + \lambda \left[ \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} - \pi_{Am}) - \pi_A (p^*_A) - \pi_B (p^*_B) \right] + \mu \left[ \pi_{Am} + \pi_{Bm} + \delta (\pi_{Ad} - \pi_{Am}) - \pi_A (p^*_A) - \pi_B (p^*_B) \right].
\]
Or equivalently:

\[
L = -\frac{1}{b_A} (p_A^*)^2 + \left(\frac{a_A}{b_A}\right) p_A^* - \frac{1}{b_B} (p_B^*)^2 + \left(\frac{a_B}{b_B}\right) p_B^*
\]

\[
+ \lambda^* \left[ \frac{1}{b_A} (p_A^*)^2 - \left(\frac{a_A + \pi}{b_A}\right) p_A^* + \frac{a_A}{b_A} \pi + 1 \left(\frac{p_B^*}{b_B}\right)^2 - \left(\frac{a_B + \pi}{b_B}\right) p_B^* + \frac{a_B}{b_B} \pi \right]
\]

\[
+ \mu^* \left[ \frac{1}{b_A} (p_A^*)^2 - \left(\frac{a_A}{b_A}\right) p_A^* + 1 \left(\frac{p_B^*}{b_B}\right)^2 - \left(\frac{a_B}{b_B}\right) p_B^* \right).
\]

The Kuhn-Tucker conditions, when all \(p_A^*\) and \(p_B^*\) must be non-negative, are given by:

\[
\frac{\partial L}{\partial p_A^*} = -\frac{2}{b_A} p_A^* + \frac{a_A}{b_A} + \lambda \left(\frac{2}{b_A} p_A^* - \frac{a_A + \pi}{b_A}\right) + \mu \left(\frac{2}{b_A} p_A^* - \frac{a_A}{b_A}\right) \leq 0;
\]

\[
p_A^* \geq 0; \quad p_A^* \frac{\partial L}{\partial p_A^*} = 0
\]

\[
\frac{\partial L}{\partial p_B^*} = -\frac{2}{b_B} p_B^* + \frac{a_B}{b_B} + \lambda \left(\frac{2}{b_B} p_B^* - \frac{a_B + \pi}{b_B}\right) + \mu \left(\frac{2}{b_B} p_B^* - \frac{a_B}{b_B}\right) \leq 0;
\]

\[
p_B^* \geq 0; \quad p_B^* \frac{\partial L}{\partial p_B^*} = 0
\]

\[
\frac{\partial L}{\partial \lambda} = \pi_A + \pi_B + \delta (\pi_A - \pi_A) - \pi_A (p_A^*) - \pi_B (p_B^*) \geq 0; \quad \lambda \geq 0; \quad \lambda \frac{\partial L}{\partial \lambda} = 0
\]

\[
\frac{\partial L}{\partial \mu} = \pi_A + \pi_B + \delta (\pi_A - \pi_A) - \pi_A (p_A^*) - \pi_B (p_B^*) \leq 0; \quad \mu \leq 0; \quad \mu \frac{\partial L}{\partial \mu} = 0.
\]

\[F \quad \text{Characterization of } S_1, S_2 \text{ and } S_3\]

Consider \(\tau = \frac{d_A}{b_A}\) which measures the degree of product substitutability. For the following different subsets of parameters' values \(\{\tau, c^E, k\}\), the solution is:

- \(Sol_1\) for all \(\delta \in [0, 1]\), when \(\{\tau, c^E, k\} \in S_1\) and this subset is given by:

<table>
<thead>
<tr>
<th>(S_1)</th>
<th>({\tau, c^E, k} \in M \cap \tilde{D})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c^E \geq c^E)</td>
<td></td>
</tr>
<tr>
<td>(k &lt; 1)</td>
<td></td>
</tr>
</tbody>
</table>
• \( \text{Sol}_1 \) for \( 0 < \delta \leq \hat{\delta} \), \( \text{Sol}_2 \) for \( \hat{\delta} < \delta \leq \delta \) and \( \text{Sol}_3 \) for \( \delta < \delta \leq 1 \), when \( \{\tau, c^E, k\} \in S_3 \) and this subset is given by:

\[
S_3 \begin{cases}
\{\tau, c^E, k\} \in M \cap \tilde{D} \\
c^E < \tau^E \\
k \leq \hat{k}
\end{cases}
\]

• \( \text{Sol}_1 \) for \( 0 < \delta \leq \hat{\delta} \) and \( \text{Sol}_2 \) for \( \hat{\delta} < \delta \leq 1 \), when \( \{\tau, c^E, k\} \in S_2 \) and this subset is given by:

\[
S_2 \begin{cases}
\{\tau, c^E, k\} \in M \cap \tilde{D} \\
c^E < \tau^E \\
k < 1 \\
\{\tau, c^E, k\} \notin S_3
\end{cases}
\]

where:

\[
c^E = \frac{(a_A \tau - 1)(2 + \tau) - (2 - \tau^2) \tilde{c}}{\tau}
\]

\[
\frac{1}{2\tau} \left[ \left(2 + \tau\right) \left(1 - \tau\right) \begin{pmatrix}
- (1 + \tau) (2 + \tau) (2 - \tau^2) \tilde{c}^2 \\
+ 2a_A (-\tau^4 + \tau^3 - 2\tau^2 - 4\tau + 8) \tilde{c}
\end{pmatrix}
+ a_A^2 (1 + \tau) (2 + \tau) (2 - \tau^2)
\right]
\]

\[
\tilde{c}^E = \frac{2 \left(-a_A (1 - \tau) (2 + \tau) - (2 - \tau^2) \tilde{c}\right)}{\tau}
\]

\[
\frac{1}{2\tau} \left[ \left(2a_A^2 \left(1 - \tau^2\right) (2 - \tau^2)^2\right) k^2 - 4a_A (1 - \tau^2) (2 - \tau^2)^2 \tilde{c} \tilde{k} - \kappa
\right]
+ \left(\tau^2 - 1\right) (4 - \tau^2)^2 \tilde{c}^2 + a_A^2 (1 - \tau^2) (4 - \tau^2)^2
\]

\[
+ 2a_A (\tau - 1) (2 + \tau) (-8 + 4\tau + 2\tau^2 - \tau^3 + \tau^4) \tilde{c}
\]

and

\[
\tilde{k} = \frac{2 \left(1 + \tau\right)^2 (4 - \tau^2)^2 \tilde{c}^2 + 2a_A^2 (1 + \tau)^2 (4 - \tau^2)^2 \tilde{c} \tilde{k} + 4a_A (1 + \tau) (2 + \tau) (8 - 4\tau - 2\tau^2 + \tau^3 - \tau^4) \tilde{c}}{2a_A (1 + \tau) (4 - \tau^2)} - \frac{\tilde{c}}{a_A}.
\]
References


