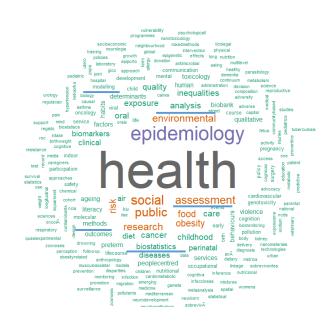


Epidemiology and the causal enquire: the role of statistics

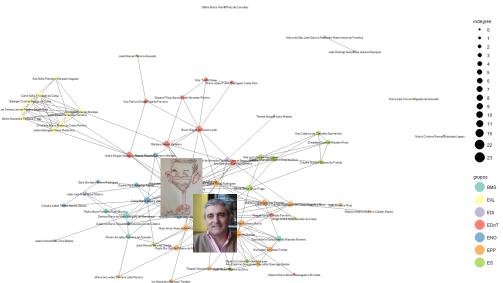
Milton Severo

Epidemiology Keywords



🗱 ISPUP

Who works with whom



Statistical association vs. causation

"Correlation does not equal causation"

Figure 14-3. Another example of association or causation. (DILBERT © 2011 Scott Adams. Used by permission of UNIVERSAL UCLICK. All rights reserved.)



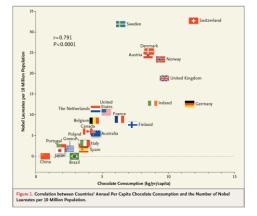




Statistical association vs. causation



"Correlation does not equal causation"



Aspects of associations to look for when assessing causality (aka causality "criteria")



Over the 20th century: Bradford Hill, Surgeon General, IARC

- Magnitude of effect
- Temporality
- Experimental evidence
- Dose-response relation
- Biological plausibility
- Consistency
- Specificity

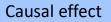
Epidemiology as observation

• In empirical science in general: Hypothesis \rightarrow Observation \rightarrow Structure (causes)

• In epidemiology:

Hypothesis about causal relation between exposure and outcome

- \rightarrow Group-level comparison of outcome frequency between exposed and unexposed groups
- ightarrow Inference about causation

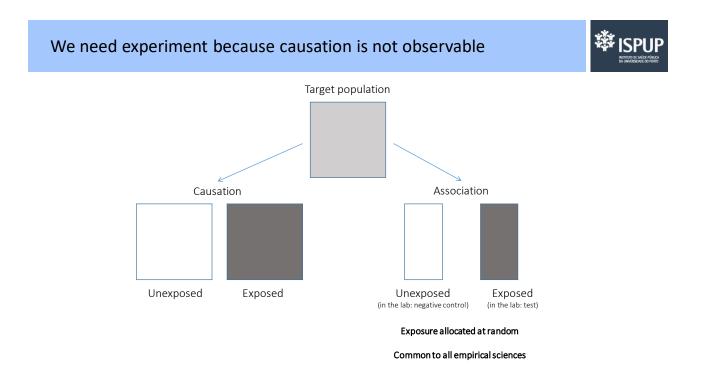


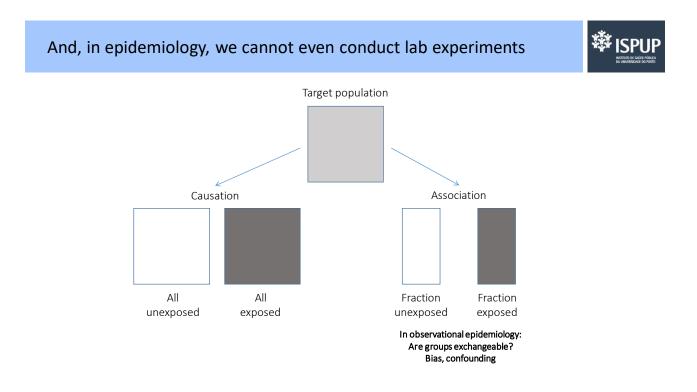
The impossible contrast between the outcome of a single unit, say an individual, if assigned the experimental treatment, and the outcome of that same individual if concurrently assigned the reference treatment.

Neyman 1923









The randomized controlled trial paradigm

- In epidemiology, randomization can be seen as a means of obtaining the observed contrast as close as possible to the counterfactual ideal.
- If we assume perfect randomization and no random error, both groups are as similar as possible with regard to measured and unmeasured factors.
- The probability of developing the outcome among the unexposed group equals the probability of developing the outcome in the exposed group had the latter not been exposed (counterfactual).

The randomized experiment paradigm for observational studies

An observational study can be seen as a conditionally randomized experiment in which:

- 1. The interventions are not assigned by the investigators
 - but hopefully are well defined
- 2. The conditional probabilities of exposure are not chosen by the investigators
 - but hopefully can be estimated from the data and are not zero
- 3. Exchangeability is not guaranteed
 - but, based on investigators' expert knowledge, is assumed conditional on measured covariates (aka confounders)

M. Hernan



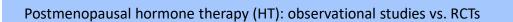




What is the effect of postmenopausal hormone therapy (HT) on coronary heart disease?

Observational vs. Experimental

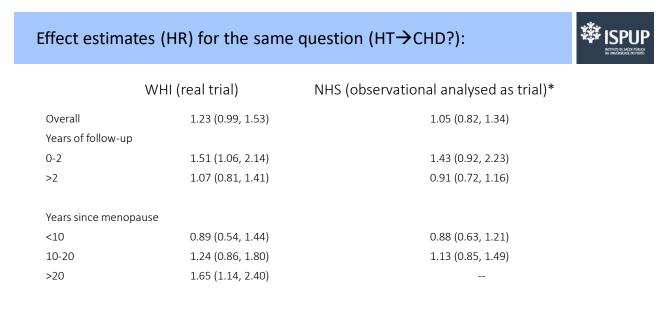
Epidemiology. 2008 Nov;19(6):766-79. doi: 10.1097/EDE.0b013e3181875e61.



- Observational studies
 - >30% lower risk in current HT users compared with never users
 - e.g., HR 0.68 in Nurses' Health Study (Grodstein et al. J Women's Health 2006)
- Randomized trials
 - >20% higher risk in initiators of HT compared with noninitiators
 - HR 1.24 in Women's Health Initiative (Manson et al. NEJM 2003)

So, studies asked different questions!

Hernan 2011.



* adjusted for potential confounders

Hernan 2011.



How to formalize and communicate causal questions and assumptions?

Causal diagrams – directed acyclic graphs (DAGs)

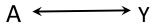
- Graphs (causal graphs or direct acyclic graphs) are considered useful for causal inference
 - · Helpful for identifying which variables to control for
 - Make assumptions explicit

The simplest DAG

• This is a direct graph, which shows that A affects Y.



• This is a undirect graph, which shows that A and Y are associated with each other



What type of study would enable this?





Question 1

- A) Cohort study
- B) Case-Control study
- C) Cross -sectional study
- D) Experimental study

Question 1

- A) Cohort study
- B) Case-Control study
- C) Cross -sectional study
- D) Experimental study

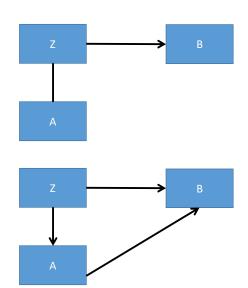


Figure 7-2. How to predict the next patient's treatment assignment in a randomized study. (PEANUTS © UFS. Reprinted by permission.)



Direct Acyclic Graphs (DAGs)

- No undirected paths
- No cycles

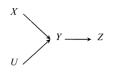


Causal diagrams – directed acyclic graphs (DAGs)



Paths

1 An example causal DAG:



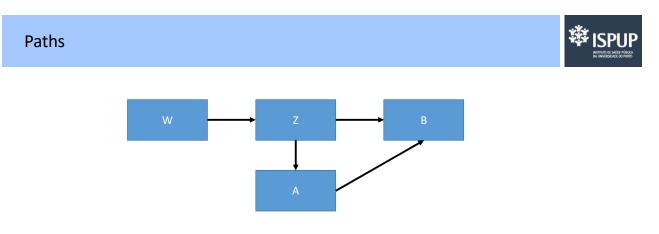
A sequence of lines (edges) between two variables, regardless of direction of arrows

Descendants

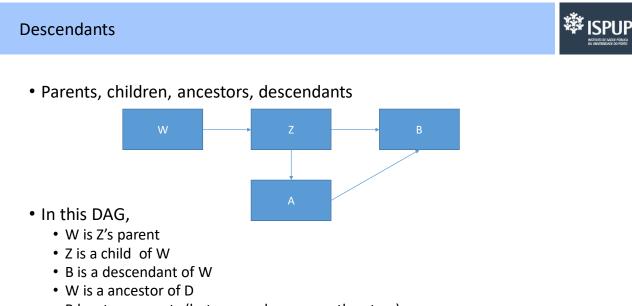
• The direct or indirect effects of a variable

Colliders

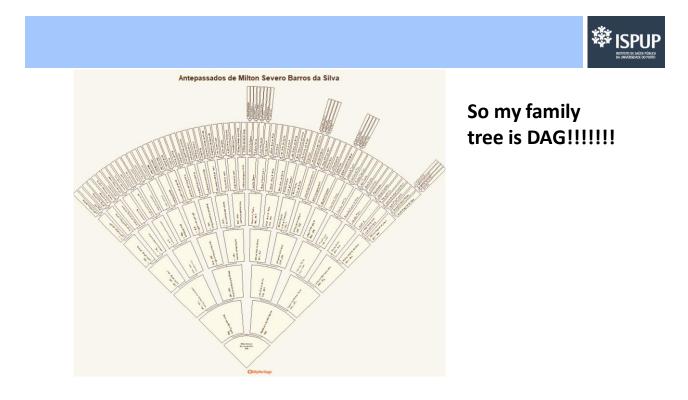
- Common effect of two variables in a path: where the arrows 'collide'.
- The two causes must both be "on the path".
- Any variable on a path that is not a collider is a "non-collider".



- A path is a way to get from one vertex to another, travelling along edges (regardless of direction of arrows)
 - There are two paths from W to B
 - W->Z->B and W->Z->A->B
 - There is one path from Z to W



• B has two parents (but we can have more than two)



DAGs

- All common causes of two or more variables in the diagram have to be explicit, regardless of whether or not they are observed
- The diagram should be parsimonious causes of only one of the vertices (variables) should not be included
- Unknown or unmeasured causes can and should be represented



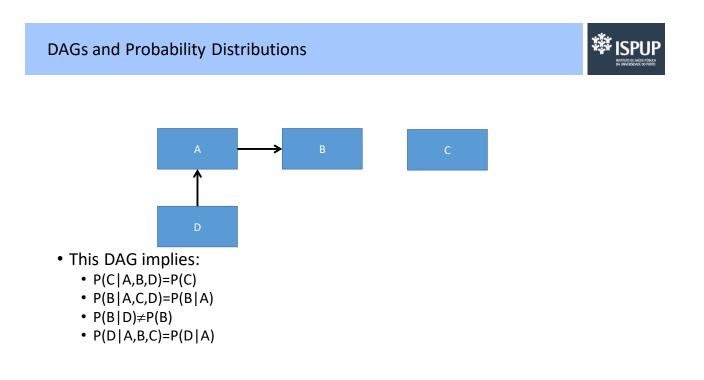
DAGs



• Where is the statistic in this drawings?

DAGs and Probability Distributions

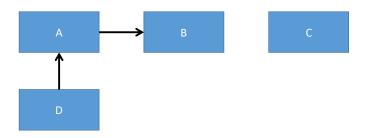
- DAGs encodes assumptions about dependencies between variables
- A DAG will tell us:
 - · Which variables are independent from each other
 - Which variables are conditionally independent from each other
 - i.e., ways that we can factor and simplify the joint distribution



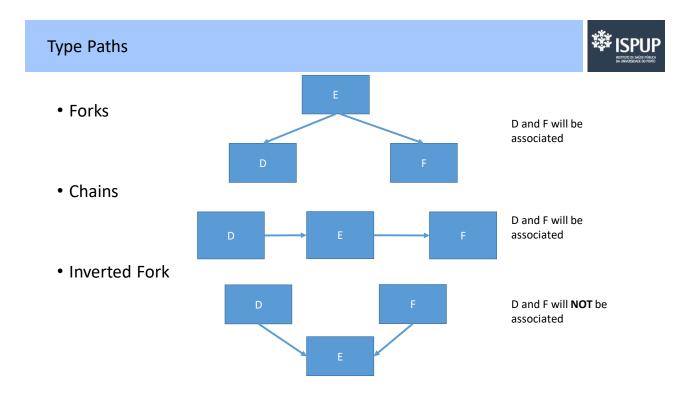
Decomposition of Joint distribution



- We can decompose the joint distribution by sequential conditioning only on sets of parents
 - Start with roots (nodes with no parents)
 - Proceed down the descendant line, always conditioning on parents



• P(A,B,C,D)=P(C)P(D)P(A|D)P(B|A)

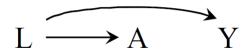


In which circumstances are A and Y causally related? And statistically associated?





Aspirin (A) and AMI (Y)



Smoking (L), Lighter (A), AMI (Y)

Does knowing A improve the prediction of Y?

Backdoor



- Backdoor paths from treatment to outcome are paths A to Y that travel through arrows going into A:
- Here, A<- L ->Y is backdoor path from A to Y.
- Backdoor path confounded relationship between A and Y

Confounding - structure



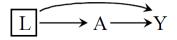


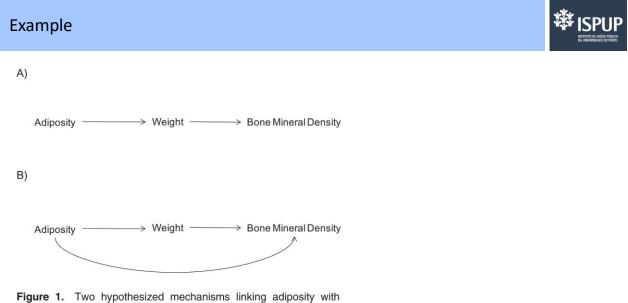
An observed statistical association between A and Y can be due to:

- A being a cause of Y

- L being a common cause of A and Y

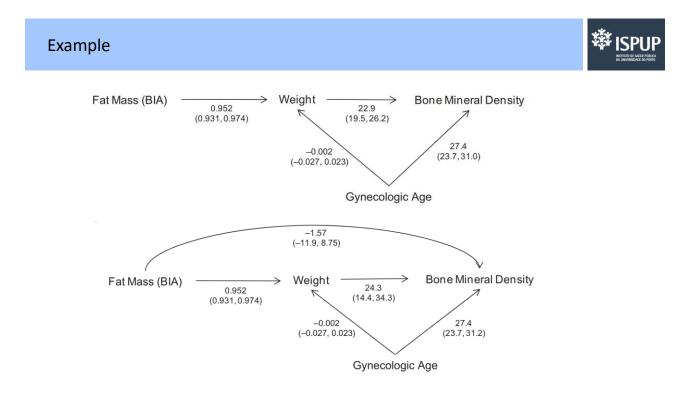
In DAGish: A \leftarrow L \rightarrow Y is an unblocked path – unless we condition on L (e.g. restrict, adjust, stratify):





bone mineral density in female adolescents: A) an overall effect totally mediated by weight and B) an overall effect with direct and indirect components.

• American Journal of Epidemiology, 2011



Example



Table 3. Estimated Indirect (Weight-Mediated) and Direct Effects^a of Adiposity on Bone Mineral Density AmongFemale Adolescents and Goodness-of-Fit Criteria for Each Tested Causal Path (see Figures 1 and 2), Porto,Portugal, 2003–2004

| | Indirect E | ffect | Direct E | Effect | Goodness of Fit | | | |
|-------------------|--|------------|--|------------------------------|--------------------------|--------------------------------------|--------------------------------------|--|
| Path ^b | Regression Coefficient (b _{st}) | 95% CI | Regression Coefficient (b _{st}) | 95% CI | Comparative Fit Index | Akaike's Information Criterion | Bayesian Information Criterion | |
| A1 | 21.8 | 18.6, 25.0 | c | | 1.000 | 14,252.492 | 14,290.813 | |
| A2 | 17.3 | 14.8, 19.8 | _ | | 1.000 | 15,460.849 | 15,499.170 | |
| B1 | 23.2 | 13.6, 32.7 | -1.57 | - 11.9 , 8 .75 | 1.000 | 14,254.389 | 14,297.500 | |
| B2 | 16.2 | 12.2, 20.2 | 1.89 | -3.13, 6.91 | 1.000 | 15,462.254 | 15,505.365 | |

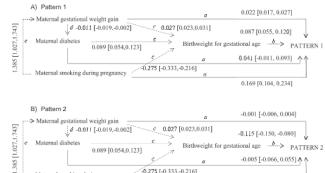
Abbreviation: CI, confidence interval.

^a Regression coefficients and 95% confidence intervals were calculated by bootstrapping using 1,000 draws.

^b Path A1: total effect of fat mass is assumed to be indirect; path A2: total effect of fat area is assumed to be indirect; path B1: total effect of fat mass is assumed to be the sum of direct and indirect components; path B2: total effect of fat area is assumed to be the sum of direct components.

^c In paths A1 and A2, it was assumed that there would be no direct association between adiposity and bone mineral density.

Example



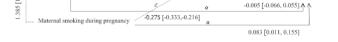


Figure 1 Causal diagram for the effects of prenatal exposures on body fat patterns identified by principal component analysis at 7-year-old children^{1,2}.

exposures (dashed arrows).

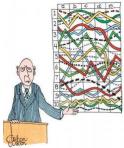
²Adjustment sets for each regression model:

Adjusted for the other two prenatal exposures, birthweight for gestational age, maternal pre-pregnancy body mass index, age and educational level at birth.

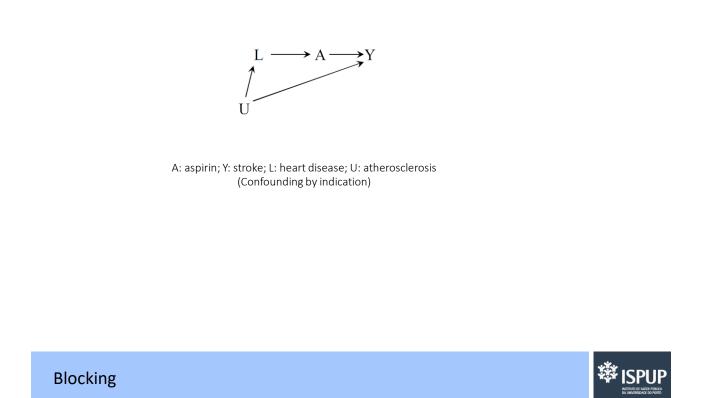
^bAdjusted for the prenatal exposures, maternal pre-pregnancy body mass index, age and educational level at birth.

^cAdjusted for the other two prenatal exposures, and maternal pre-pregnancy body mass index. ^dAdjusted for maternal pre-pregnancy body mass index and age at birth.

eAdjusted for maternal pre-pregnancy body mass index, age and educational level at birth.



"Ill pause for a moment so you can let this information sink in."



What are the confounders and how to deal with confounding?

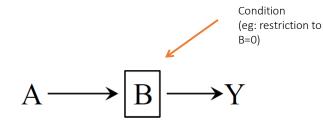
- Paths can be blocked by conditioning on variables (vertices) in the path
- Consider the path:



- If we condition on E (a node in the middle of chain), we block the path from D to ${\rm F}$

Is there an association between A and Y in each level of B? If we know B, does knowing A improve the prediction of Y?





Aspirin (A), platelet aggregation (B), AMI (Y)

A and Y are marginally associated but conditionally independent, given B



- Associations on fork (confunding) can also be block
- Consider the path:
- A<-G->B
- If we condition on G, this path from A to B is blocked.



Example 2

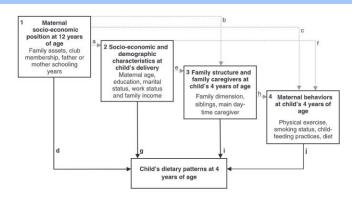


Fig. 1. Theoretical framework of family and maternal determinants of children's diet. This figure depicts the theoretical framework defined for analysis in the present study adapted prom previously published models (UNICEF, 1990; Victora et al. 1997; Davison & Birch, 2001; Solar & Irwin, 2010). Arrows represent theoretical causal relationships between determinants of children's dietary patterns. Dashed grey lines represent possible indirect effects in the pathway between levels of determinants. Solid black lines represent the direct effects of factors, after adjustment for determinants in preceding levels that is not mediated by subsequent ones, but that may be explained by other factors (unknown or unmeasured). I. Socio-economic position at mothers' 12 years may exert an effect on children's diet through socio-economic and demographic characteristics at child's delivery (a), through its influence on subsequent family characteristics (b), through maternal behaviours (c) and/or through unknown or unmeasured determinants (d). 2. Socio-economic and demographic characteristics at child's delivery may have an effect on children's diet through subsequent family characteristics (e), through maternal behaviours (f) and/or through unknown or unmeasured factors (g). 3. Family characteristics at child's 4 years of age may have an effect on children's dietary patterns through their influence on maternal behaviours (h) and/or through unknown or unmeasured determinants (i). 4. In this conceptual framework, maternal behaviours would then influence children's dietary patterns directly and/or through other unknown or unmeasured factors (j). In the present study, we were particularly interested in the overall effects and the direct effects (highlighted in bold, d, g, i and j).

Maternal & Child Nutrition, 2017

Table 3. Multivariate analysis of the associations of maternal and family characteristics with dietary patterns of 4-year-old children, n = 3422*

| | n | Model 1 [†] | | Model 2 [†] | | Model 3 [†] | | Model 4 [†] | |
|----------------------------------|---------------|--------------------------------------|-----------------------|------------------------------------|------------------------------------|--|------------------------------------|--|-----------------------|
| | | $\mathrm{EDF}^{\$}$ | Snacking [‡] | EDF [‡] | Snacking [‡] | EDF^{\ddagger} | Snacking [‡] | EDF [‡] | Snacking [‡] |
| | | n = 1400 | <i>n</i> = 484 | <i>n</i> = 1400 | <i>n</i> = 484 | <i>n</i> = 1400 | <i>n</i> = 484 | n = 1400 | <i>n</i> = 484 |
| | | | | | | | | | |
| Socio-economic position at m | | ears | | 2 | ъ. | | | 2 | 2 |
| High Intermediate | 894 1735 | 1 53 (1 35 1 03) | 1.56 (1.20-2.02) | 1 1.04 (0.85–1.26) | 1 1.09 (0.82–1.44) | 1 1.04 (0.86–1.26) | 1 1.09 (0.82–1.44) | 1 1.03 (0.83–1.27) | 1 1.08 (0.81–1.45 |
| Low | 793 | 1.52 (1.27-1.83) 1.76 (1.42-2.18) | 1.56 (1.20-2.02) | 1.04(0.85-1.26) 1.10(0.86-1.41) | 1.09(0.82-1.44) 1.06(0.82-1.49) | 1.04 (0.86 - 1.26) 1.10 (0.86 - 1.41) | 1.09(0.82-1.44) 1.07(0.75-1.51) | 1.03 (0.83 - 1.27) 1.19 (0.91 - 1.55) | 1.08 (0.81–1.45 |
| Socio-economic and demogra | | | | 1.10 (0.80–1.41) | 1.06 (0.82–1.49) | 1.10 (0.80–1.41) | 1.07 (0.75–1.51) | 1.19 (0.91–1.55) | 1.12 (0.79-1.60 |
| Maternal age | ipnic charac | teristics at child's de | envery | | | | | | |
| >29 years | 1997 | | | ă. | | 1 | | ¥ | 1 |
| | 993 | | | 1.28 (1.07-1.52) | 1.26 (0.99–1.60) | 1.38 (1.15-1.66) | 1.14 (0.88–1.46) | 1 | 0.97 (0.75-1.25 |
| 25–29 years | | | | | | | | | |
| <25 years | 432 | | | 2.17 (1.67-2.84) | 1.63 (1.14-2.33) | 2.47 (1.87-3.28) | 1.40 (0.96-2.04) | 1.80 (1.34-2.44) | 1.07 (0.73-1.58 |
| Maternal education | | | | | | | | | |
| >12 years | 1050 | | | 1 | 1 | 1 | 1 | 1 | 1 |
| 10-12 years | 990 | | | 1.91 (1.54-2.35) | 1.61 (1.19-2.18) | 1.87 (1.51-2.31) | 1.67 (1.23-2.27) | 1.51 (1.20-1.90) | 1.44 (1.05-1.98 |
| ≤9 years | 1382 | | | 2.87 (2.29-3.59) | 2.81 (2.06-3.83) | 2.76 (2.19-3.47) | 3.02 (2.20-4.15) | 2.19 (1.70-2.81) | 2.55 (1.82-3.55 |
| Maternal work status | | | | | | | | | |
| Working | 2742 | | | 1 | 1 | 1 | 1 | 1 | 1 |
| Not working | 680 | | | 1.07 (0.87-1.30) | 0.99 (0.76-1.30) | 1.06 (0.86-1.29) | 1.03 (0.78-1.35) | 1.00 (0.80-1.24) | 0.99 (0.74-1.31 |
| Family at child's 4 years | | | | | | | | | |
| Family dimension | | | | | | | | | |
| <4 persons | 1434 | | | | | 1 | 1 | 1 | 1 |
| 4 persons | 1456 | | | | | 0.96 (0.71-1.29) | 0.74 (0.49-1.13) | 0.91 (0.66-1.25) | 0.71 (0.46-1.09 |
| >4 persons | 532 | | | | | 0.83 (0.59-1.15) | 0.74 (0.47-1.17) | 0.80 (0.56-1.15) | 0.72 (0.45-1.16 |
| Child's siblings | | | | | | | | | |
| No siblings | 1613 | | | | | 1 | 1 | 1 | 1 |
| Older and younger | 84 | | | | | 1.09 (0.61-1.95) | 0.77 (0.32-1.81) | 1.40 (0.75-2.61) | 0.92 (0.38-2.21 |
| Only younger | 419 | | | | | 1.10 (0.77-1.57) | 1.43 (0.88-2.32) | 1.38 (0.95-2.03) | 1.67 (1.01-2.73 |
| Only older | 1306 | | | | | 1.40 (1.04-1.89) | 0.91 (0.60-1.38) | 1.67 (1.21-2.30) | 1.02 (0.66-1.58 |
| Main daytime caregiver | | | | | | | | | |
| Not family ⁸ | 2951 | | | | | 1 | 1 | 1 | 1 |
| Parent ⁸ | 140 | | | | | 1.12 (0.73-1.73) | 1.63 (0.98-2.72) | 1.36 (0.86-2.16) | 1.84 (1.09-3.10 |
| Other family member [§] | 331 | | | | | 1.23 (0.93-1.62) | 1.29 (0.91-1.84) | 1.14 (0.85–1.53) | 1.23 (0.86-1.77 |
| Maternal characteristics at ch | ild's 4 years | | | | | | | | |
| Physical exercise | | | | | | | | | |
| Practitioners | 667 | | | | | | | 1 | 1 |
| Non-practitioners | 2755 | | | | | | | 1.10 (0.89-1.36) | 1.21 (0.91-1.62 |
| Smoking status | | | | | | | | | |
| Non-smokers | 2705 | | | | | | | 1 | 1 |
| 1-10 cigarettes/day | 459 | | | | | | | 1.22 (0.95-1.56) | 1.11 (0.80-1.54 |
| >10 cigarettes/day | 258 | | | | | | | 1.09 (0.79-1.50) | 0.86 (0.55-1.34 |

Table 3. (Continued)

| | п | Model 1^{\dagger} | | Model 2 [†] | | Model 3 [†] | | Model 4 [†] | |
|------------------------------------|------|---------------------|-----------------------|----------------------|-----------------------|----------------------|-----------------------|----------------------|-----------------------|
| | | EDF^{\ddagger} | Snacking [‡] | EDF^{\ddagger} | Snacking [‡] | EDF^{\ddagger} | Snacking [‡] | EDF^{\ddagger} | Snacking [‡] |
| | | <i>n</i> = 1400 | <i>n</i> = 484 | <i>n</i> = 1400 | <i>n</i> = 484 | <i>n</i> = 1400 | <i>n</i> = 484 | <i>n</i> = 1400 | <i>n</i> = 484 |
| Dietary score | | | | | | | | | |
| 4th quartile (>22 Pt) | 607 | | | | | | | 1 | 1 |
| 3rd quartile (20-22 Pt) | 899 | | | | | | | 2.69 (2.02-3.58) | 1.59 (1.13-2.25) |
| 2nd quartile (17-19 Pt) | 992 | | | | | | | 6.13 (4.62-8.12) | 2.41 (1.71-3.40) |
| 1st quartile (<16 Pt) | 924 | | | | | | | 9.94 (7.35-13.44) | 4.21 (2.94-6.05) |
| BMI | | | | | | | | | |
| $<25.0 \text{ kg/m}^2$ | 1650 | | | | | | | 1 | 1 |
| ≥25.0 kg/m ² | 1772 | | | | | | | 1.16 (0.98-1.38) | 0.95 (0.76-1.19) |
| Child-feeding patterns | | | | | | | | | |
| Perceived monitoring | 3422 | | | | | | | 0.84 (0.77-0.91) | 0.89 (0.80-0.99) |
| Restriction | 3422 | | | | | | | 0.85 (0.78-0.93) | 0.88 (0.78-0.98) |
| Pressure to eat | 3422 | | | | | | | 0.95(0.87 - 1.04) | 1.07 (0.95-1.20) |
| Nagelkerke's <i>R</i> ² | | | 0.08 | | 0.14 | | 0.15 | 0.3 | |
| LRT P-value | | | <0.001 | | <0.001 | | < 0.001 | <0. | 001 |

BMI, body mass index; EDF, energy-dense foods dietary pattern; Nagelkerke's R^2 , Nagelkerke's R^2 -squared (Nagelkerke, 1991); Pt, points; LRT, likelihood ratio test. *Statistically significant associations are highlighted in bold. [†]Blocks of variables (socio-economic position at mothers' 12 years of age; maternal socio-economic and demographic characteristics at child's delivery; family characteristics at child's 4 years) were added sequentially into the analysis. Models are adjusted for child's characteristics (sex; daily screen time; weekly time spent practicing physical exercise; and exact age). [†]Beference category is the Healthier dietary pattern (n = 1538), not shown to avoid redundancy. [§]Not family, kindergarten and nannies; parent, mostly mothers (95%); other family member, mostly grandparents (96%).

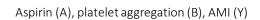
Question 2

- Socio-economic position at mother 12 years is considering that the DAG is correctly defined:
 - A) not associated to dietary pattern at 4 years
 - B) associated to dietary pattern at 4 years but is not causal effect
 - C) Causal effect of dietary pattern at 4 years



Confounders cannot be intermediate steps

A٠

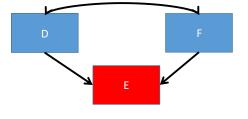


В

≻Y



- The opposite situation occurs if collider is conditioned on
- Consider the path:
- A->G<-B
- Here, A and B are not associated via this path
- However, conditioning on G induces an association between A and B
 - Opens door between A->B







Blocking

```
> a<-rnorm(100)
> b<-rnorm(100)
> g<-a+b+rnorm(100)
 > cor.test(a,b)
                 Pearson's product-moment correlation
data: a and b
t = -0.45003, df = 98, p-value = 0.6537
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
-0.2396932 0.1523641
sample estimates:
corr

                  cor
 -0.04541314
 > mod<-lm(scale(a)~scale(b)+scale(g))</pre>
 > summary(mod)
 Call:
 lm(formula = scale(a) ~ scale(b) + scale(g))
 Residuals:
Min 1Q Median 3Q Max
-1.44512 -0.50558 -0.03178 0.44389 1.34114
 Coefficients:

        Coefficients:
        Estimate Std. Error t value Pr(>|t|)

        (Intercept) -7.799e-18
        6.611e-02
        0.000
        1

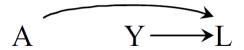
        scale(b)
        -4.881e-01
        7.703e-02
        -6.336
        7.39e-09
        ***

        scale(g)
        8.750e-01
        7.703e-02
        11.359
        < 2e-16</td>
        ***

 signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.6611 on 97 degrees of freedom
Multiple R-squared: 0.5717, Adjusted R-squared: 0.5629
F-statistic: 64.75 on 2 and 97 DF, p-value: < 2.2e-16
```

Collider: In which circumstances are A and Y causally related? And statistically associated?

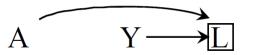




Genotype (A), smoking (Y), AMI (L)

Does knowing A improve the prediction of Y?

Adjusting for a collider causes bias



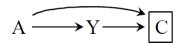
If L=1: in individuals with AMI, knowing that they do not smoke modifies the probability of them having a risk genotype

Genotype (A), smoking (Y), AMI (L)

Selecting L=1 (AMI present) originates an open path: $A \rightarrow L \leftarrow Y$, i.e., a source of statistical association – but we are only interested in the causal $A \rightarrow Y$, which is null

RR causal ≠ RR association

Selection bias in study design – conditioning on a common effect



A: Folic acid; Y: congenital heart defect; C: fetal death

Selecting C=0 (live births) originates 2 open paths: $A \rightarrow Y$ and $A \rightarrow C \leftarrow Y$, i.e., two sources of statistical association – but we are only interested in the causal $A \rightarrow Y$

RR causal ≠ RR association How to solve?



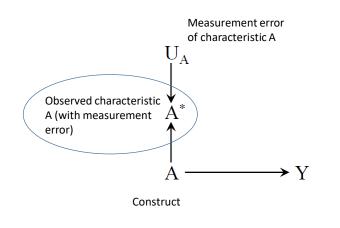
 \mathbf{Y}_1

→_{**π**}Y₂

U: unmeasured organic condition Conditioning on Y1 unblocks the path $A \rightarrow Y1 \leftarrow U \rightarrow Y2$, i.e. induces a statistical

association when there is no causal relation from A to Y2

Information/misclassification bias











3.1 | Mixed-effects polynomials

In the mixed-effects polynomial model, the power (p) of each term corresponds to the order of the term (d) ($\mathbf{p} = \mathbf{d}$). For example, the general expression for the model of weight adjusted for height, including three terms, is as follows:

$$\ln\left(w_{ij}\right) = \beta_0 + \beta_1 h_{ij}^{\ 1} + \beta_2 h_{ij}^{\ 2} + \beta_3 h_{ij}^{\ 3} + b_{0i} + b_{1i} h_{ij}^{\ 1} + b_{2i} h_{ij}^{\ 2} + b_{3i} h_{ij}^{\ 3} + \varepsilon_{ij},\tag{1}$$

where w_{ij} and h_{ij} is the weight (g) and the height (m), respectively, for individual *i* at time *j*; β are the fixed terms; and **b** are the random effects with N(0, σ_d), d = 1, 2, and 3.

3.2 | Mixed-effects fractional polynomials

The mixed-effects fractional polynomial smoothing method²³ is a general case of the mixed polynomial where the powers can be different from the order of the terms ($\mathbf{p} \neq \mathbf{d}$). The mixed-effects fractional polynomial with three terms has the following general expression, as an example, for the model parameterization of weight adjusted for height. If all p_d are different for d = 1, 2, and 3,

$$ln(w_{ij}) = \beta_0 + \beta_1 h_{ij}^{(p_1)} + \beta_2 h_{ij}^{(p_2)} + \beta_3 h_{ij}^{(p_3)} + b_0 + b_1 h_{ij}^{(p_1)} + b_2 h_{ij}^{(p_2)} + b_3 h_{ij}^{(p_3)} + \varepsilon_{ij},$$
(2)

Statistics in medicine, 2019

3.3 | Linear-splines mixed effects

The linear-splines mixed-effects²⁴ smoothing method with three linear splines has the following expression, as an example, for the of weight adjusted for height parameterization:

$$\ln(w_{ij}) = \beta_0 + \beta_1 (h_{ij} - h_0)_+ + \beta_2 (h_{ij} - k_1)_+ + \beta_3 (h_{ij} - k_2)_+ + b_{0i} + b_{1i} (h_{ij} - h_0)_+ + b_{2i} (h_{ij} - k_1)_+ + b_{3i} (h_{ij} - k_2)_+ + \epsilon_{ij},$$
(4)

17.14

0.81

5.84

1.11

7.69

6.82

0.75

5.70

0.77

5.82

weight adjusted for different covariates **Covariate in the Model Fit Indices** Height Age Polynomial RSE 4.35 0.91

RAE

RSE

RAE

RSE

RAE

TABLE 4 Fit indices of different statistical models of natural log of

RSE: relative squared error; RAE: relative absolute error.

Fractional Polynomial

Linear splines

1.7

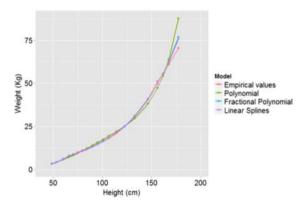
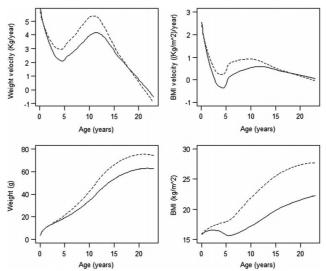


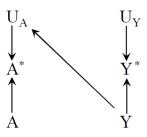
FIGURE 2 Growth curves for weight-for-height estimated from the three different statistical models (polynomial, fractional polynomial and linear splines), compared to the empirical mean values of the sample [Colour figure can be viewed at wileyonlinelibrary.com]



International Journal of Obesity **2015**

Figure 1. Weight velocity (top left), BMI velocity (top right), mean values of predicted weight (bottom left) and predicted BMI (bottom right) throughout age, according to the two trajectories identified. Solid line: 'Average BMI growth'; dashed line: 'Higher BMI growth'.





A: drug; Y: dementia (self-reported medication asked by interview) or A: alcohol intake during pregnancy; Y: birth defect

What are the traditional names of these biases?

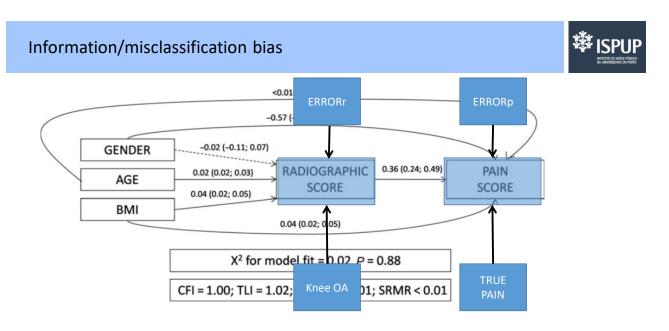
Information/misclassification bias

|--|

| | Knee pain | Radiogra | phic score | Crude | Adjusted odds ratio** | |
|--------------|--------------|------------|------------|------------------------|------------------------|--|
| | | KL < 2 | KL ≥ 2 | Crude | | |
| | score | n (%) | n (%) | odds ratio* | | |
| | -1 | 222 (70.3) | 128 (53.1) | 1 (reference category) | 1 (reference category) | |
| | 0 | 38 (12.0) | 24 (10.0) | 1.10 (0.63; 1.91) | 0.85 (0.46; 1.58) | |
| BDI < 14 | 1 | 27 (8.5) | 21 (8.7) | 1.35 (0.73; 2.48) | 1.03 (0.51; 2.07) | |
| | 2 | 24 (7.6) | 40 (16.6) | 2.89 (1.67; 5.01) | 2.28 (1.21; 4.30) | |
| | 3 | 5 (1.6) | 28 (11.6) | 9.71 (3.66; 25.78) | 5.37 (1.90; 15.18) | |
| | -1 | 20 (43.5) | 13 (21.7) | 1 (reference category) | 1 (reference category) | |
| | 0 | 4 (8.7) | 4 (6.7) | 1.54 (0.33; 7.26) | 1.68 (0.35; 8.18) | |
| $BDI \ge 14$ | 1 | 6 (13.0) | 4 (6.7) | 1.03 (0.24; 4.35) | 1.14 (0.25; 5.14) | |
| | 2 | 8 (17.4) | 20 (33.3) | 3.85 (1.31;11.29) | 3.60 (1.17; 11.07) | |
| | 3 | 8 (17.4) | 19 (31.7) | 3.65 (1.24; 10.78) | 2.73 (0.84; 8.86) | |

Table 3 Association between radiographic score and knee pain score, according to depressive symptoms

*Crude odds ratio for radiographic OA (KL \geq 2); **Adjusted odds ratio for age, body mass index (BMI) and gender.



International Journal of Rheumatic Diseases, 2017

Structural classification of bias

- Two variables are statistically associated when
 - One is a cause of the other ← our aim!
 - They share common causes confounding
 - They share effects that have been conditioned on selection bias
 - There is differential measurement error information/misclassification bias

Bias – any structural association between exposure and outcome that does not arise from the causal effect of the exposure on the outcome

Rules for d-separation

- A path is d-separated by set of variables C if:
 - It contains a chain (D->E->F) and the middle part is in C
 - It contains a fork (D<-E->F) and the midle part is in C
 - It contains na inverted fork (D->E<-F) and the midle part is not in C, nor any descendants of it

D-separation, Pearl 1995 ("Moralization", Lauritzen 1990)

A path is a sequence of edges that connect two nodes in a graph. The path is said to be open or closed according to the following rules:

- 1. If no variable has been conditioned on, a path is blocked if and only if two edges collide along the path: $L \rightarrow A \rightarrow Y$ is an open path but $A \rightarrow Y \leftarrow L$ is a closed path: Y is called a collider
- 2. Any path that contains a non-collider that has been conditioned on is blocked: $L \rightarrow A \rightarrow Y$
- 3. Conditioning on a collider unblocks the path: A → Y ← L
- 4. Conditioning on the descendant of a collider unblocks the path





Rules for d-separation



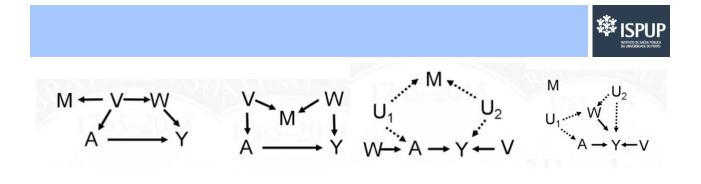


Figure 20-3. One view of the seemingly endless stream of reported risks confronting the public. (Jim Borgman. The Cincinnati Enquirer. 1997. Reprinted with special permission of King Features Syndicate.)

Variable selection

- The usual criterion would be adjusting for all variables (Criterion 1)
- The disjunctive cause criterion (VanderWeele 2011) (Criterion 2)
 - Control for all (observed) causes of exposure, outcome or both
- Researchers do not know the whole graph, but rather, the list of variables that affect the exposure or outcome





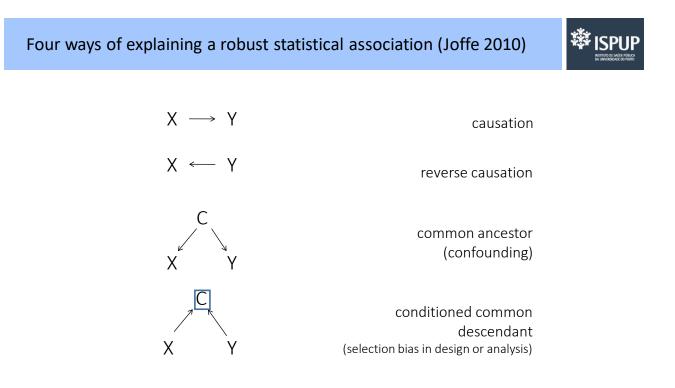
| | DAG 1 | DAG 2 | DAG 3 | DAG 4 |
|-------------|-------|-------|-------|-------|
| Criterion 1 | YES | YES | NO | NO |
| Criterion 2 | YES | YES | YES | NO |

Confounding – marginal and conditional independence



It is possible to identify causal relations when, between the exposure and the outcome,

- There are no common causes (RCTs)
- There are common causes but enough variables were measured that allow for blocking all backdoor paths – in that case it is said that there is confounding but no unmeasured confounding → Adjust, Stratify, Mactching, Standardize, etc.



Causal diagrams

By representing

- Previous knowledge
- Assumptions

And applying a set of logical rules

It is possible to

- Understand the extent to which observed data are consistent with the causal model
- Predict expected statistical associations
- Detect logical problems and contradictions in data analysis

Acknowledge



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Epidemiology and the causal enquire: the role of statistics

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